

Lab 4 Robots in a Plane (Static Obstacle Avoidance)
15-424/15-624 Foundations of Cyber-Physical Systems

Test Due Date: Friday, 10/25/13, worth 20 points

Final Due Date: Friday, 11/1/13, worth 80 points

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1. In this lab you will design a controller which may move freely (non-deterministically) on a plane, rather than just around fixed to a circular track. The robot should be able to move anywhere, but it must always avoid a single, static (not moving) obstacle.

Modeling the free motion of a robot in 2D can be thought of as an extension of lab3, but now with discrete control of steering as well as acceleration. When steering is changed, you might think of it as the robot switching from one circular track to another, but the new track must be tangent to the old one at the position of the robot so that the robot can maintain its position, direction and velocity. The new track's radius and whether the track is on the left or the right of the car (in other words, whether the car is traveling clockwise or counter-clockwise around the track) may change at each discrete transition. To help you visualize what is happening, we have created this youtube video: http://youtu.be/C_pyRQT6bBw.

To help you get started with this assignment, you may download a template for the key file here: <http://symbolaris.com/course/fcps13/template4.key>. This file contains the annotations which are required for simulation. We've included some variables which are not strictly necessary for modeling or proving, but which you may uncomment. If you want to add more variables about the state of the system, please post your request on Piazza so that we can create a standard naming scheme.

- You **may not** have discrete changes in the position, direction, or linear velocity of the robot (or any other variable which would implicitly cause such a discrete change).
- You may (and should) discretely control the track radius (which can be negative to change the direction of travel from clockwise to counter-clockwise) and acceleration of the robot.
- The robot should have a non-deterministic controller (i.e. it should be able to drive anywhere that does not cause it to come too close to the obstacle).
- The robot should be time-triggered.
- Since you are now freely moving in 2D, your robot now has a shape, which can be over-approximated by a circle of radius $r > 0$.
- The obstacle also can be over-approximated as a circle of radius $obsr > 0$ (hint: you may still model your system using points, so long as a sufficient buffer is kept between the robot's point and the obstacle's point).
- The robot should always have a non-zero turning radius (i.e. it can't spin in place).

Suggestions and hints:

- We recommend that you use braking to enforce safety, and leave steering entirely non-deterministic.
- You may also simplify your model to represent an infinitesimal point (x, y) for the position of the robot and point $(obsx, obsy)$ as the point of the obstacle **provided** you ensure that the two never get within a symbolic *buffer* distance of each other.
- Using guarded non-deterministic assignment will be very useful in this problem. You may use it to choose the radius *trackr* of the conceptual "track" that the robot is moving on.
- If you found using the Max function to be difficult during proving, you may find it easier to reformulate those statements using disjunction instead.

- Use the provided template (available at <http://symbolaris.com/course/fcps13/template4.key>).

1.1 [test] Fill in the missing parts of the provided template to model the hybrid program above and verify that it is safe. Submit this file as `test_username.key`.

1.2 [final] Use KeYmaera to prove that your hybrid program is safe. Submit the resulting proof and corresponding `.key` file as `final_username.proof` and `final_username.key`.

2. **Question:** What if we let the obstacle move with velocity less than or equal to *obsv*? What if your robot has a top speed that's less than *obsv*? What if instead of moving on an unbounded 2D plane your robot is constrained to a bounded space? In these cases, even if your robot is stopped, the obstacle could still hit you! Propose and discuss a few possible safety properties for these new scenarios which, if proved, would guarantee that your robot exhibits reasonable behavior even in the presence of an unreasonable obstacle. What are the pros and cons of each of your proposed safety properties? Submit your answer to this question in `lab4_username.txt`.

3. **Submission checklist.**

Test submission (Due 10/25):

`test_username.key`

Final submission (Due 11/1):

`final_username.key`

`final_username.proof`

`lab4_username.txt`