Orbital Mechanics of Gravitational Slingshots

Adam Moran and John Mann

15-424: Foundations of Cyber-Physical Systems

Outline

- Overview
- The Model
- The Proof
- Limitations
- Future Work

Gravity Slingshots

Background

- A gravity slingshot is a maneuver that results in an energy transfer between an approaching spacecraft and large celestial body.
  - Can be used to speed up, slow down, and redirect vehicles.
- When the spacecraft approaches, it gains speed as it falls towards the planet, then gains enough speed to surpass escape velocity (V_e)

Motivation

- Fuel = money for space travel.
  - Bringing more fuel into orbit requires even more fuel to lift the fuel.
- Gravity slingshots can save a lot of fuel, and therefore make deep-space missions more cost-effective.

The Model

r_{planet}	radius of planet
r_{orbit}	radius of orbit
$h_{atmosphere}$	atmosphere
Θ	current angle
Θ_{sling}	desired angle
v	current velocity
v_e	escape velocity
x	scale factor
c	cosine
s	sine

Safety $r_{planet} + h_{atmosphere} \leq r_{orbit}$

Efficiency $(\Theta \leq \Theta_{sling}) \rightarrow (v \leq v_e)$

Model c' = -s,

s' = c,

v' = x*thrust + c,

theta' = v/orbitr

Putting it together

(/* init */) $\rightarrow$

[

{ thrust := *; ?(thrust < v_e - v); }

{

c’ = -s,

s’ = c,

v' = x*thrust + c,

Θ’ = v/r_{orbit} ,

t’ = 1

}

](

$r_{planet} + h_{atmosphere} \leq r_{orbit} \quad \wedge$

$(\Theta \leq \Theta_{sling}) \rightarrow (v \leq v_e)$

)

} Model

} Safety and Efficiency

Putting it together

(/* init */) $\rightarrow$

[

  { thrust := *; ?(thrust < v_e - v); }

  {

    $c' = -s$,

    $s' = c$,

    $v' = x*thrust + c$,

    $\Theta' = v/r_{orbit}$,

    $t' = 1$

  }

](

  $r_{planet} + h_{atmosphere} \leq r_{orbit} \quad \wedge$

  $(\Theta \leq \Theta_{sling}) \rightarrow (v \leq v_e)$

)

Proof: Key Invariants

$c^2 + s^2 = 1$

$r_{planet} + 150 \leq r_{orbit}$

$v^2 \leq \left(\frac{2GM}{r_{orbit}}\right)$

Limitations

In our model, r_{orbit} is kept constant while the spacecraft is under acceleration.

Normally, r_{orbit} will increase as velocity increases.

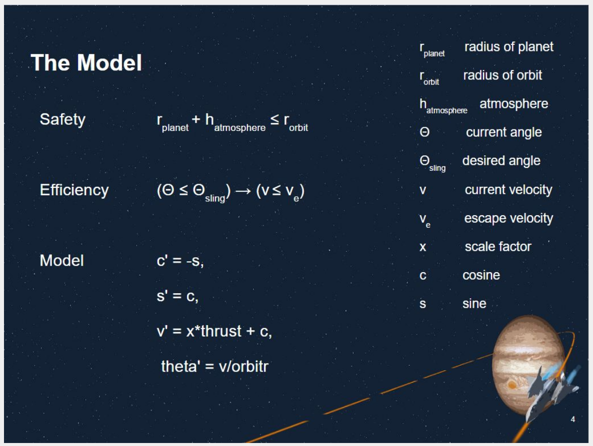
It is physically possible to thrust such that the orbital radius is maintained, but speed is increased. However, such an engine burn requires *much* more fuel than a simple tangent one.

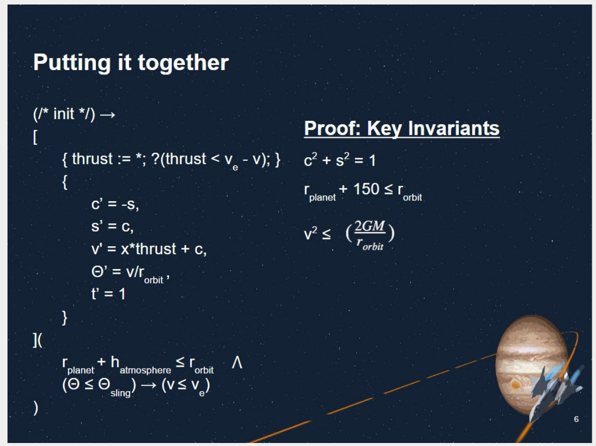
Thankfully, this is not a problem for our no-mass, infinite-fuel spacecraft.

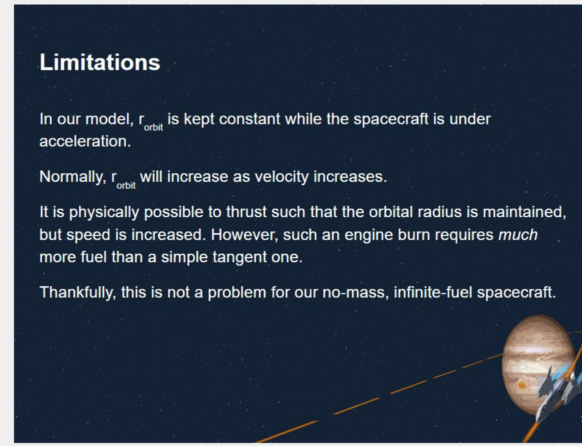
Future Work

- Make the spacecraft more realistic.
    - Give it a dry mass and wet mass?
    - Have its acceleration change according to rocket equation physics?
- Improved orbital physics.
    - In a more realistic and fuel-efficient simulation, the orbital radius would increase as the velocity of the spacecraft increases.

Wrap Up







Questions?