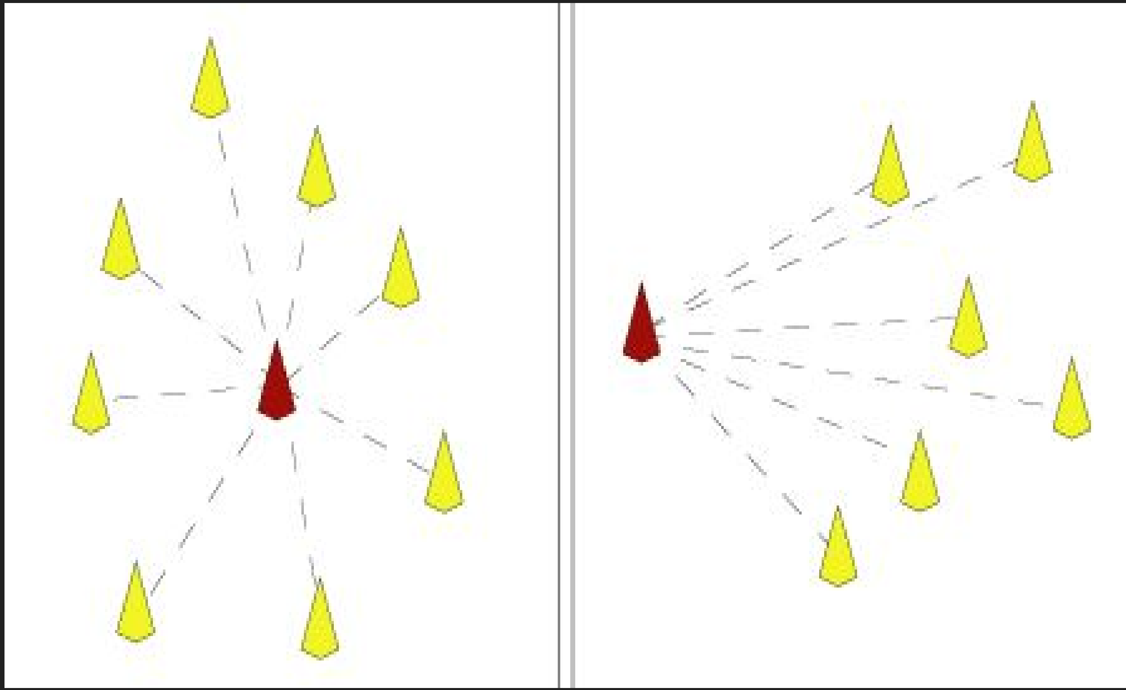
Flock on the March

A Meta-Proof of a Meta-Model

By Zihan Zhou
QCS, Class of 2017

What am I modeling?

- Boids
  - Infinite many
  - Same destination
  - Not a transportation protocol
- Obstacles
  - Infinite many
  - Arbitrary shape
  - Arbitrary size
  - Pass some
    - Distant enough

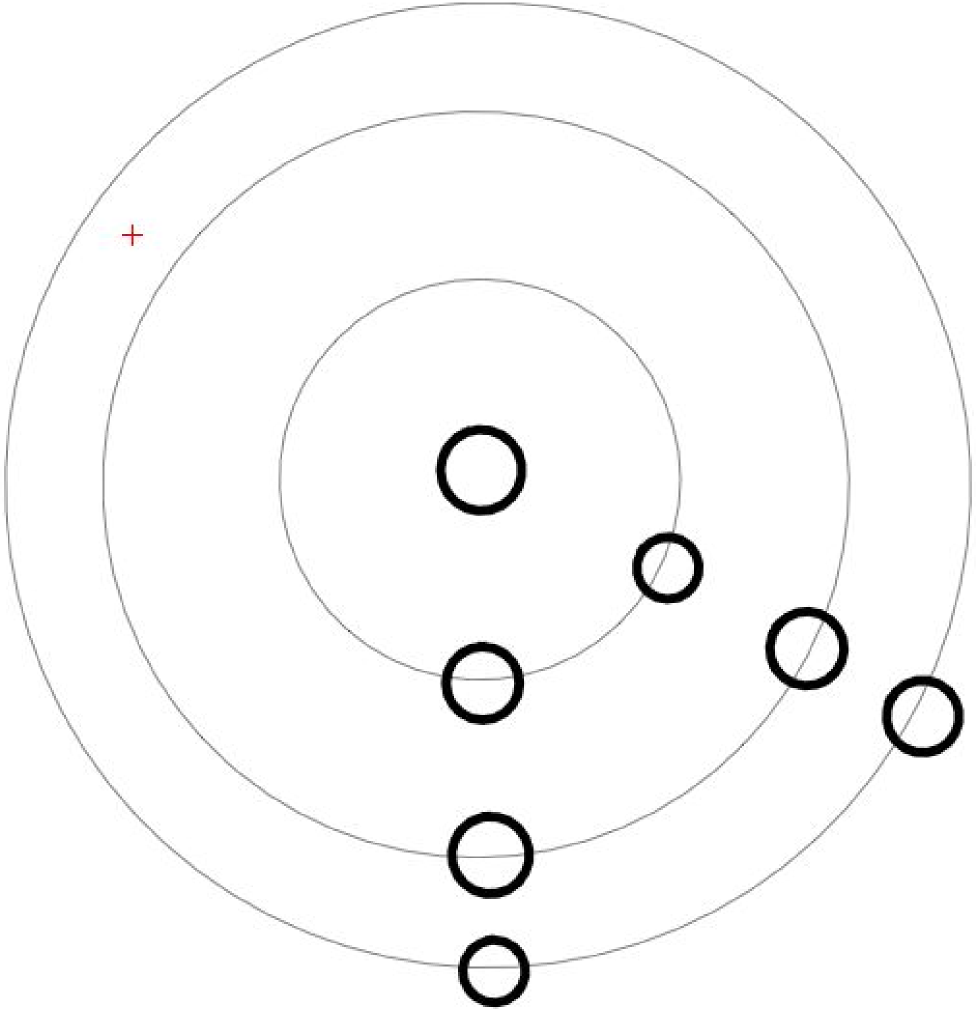


Project Highlights

- Without Distributed DL!
  - This model is constructed in a certain way
- Framework , Modular
  - Control
  - Proof
- Analogy
  - Functions
  - Specs for functions
    - Contracts

Leader-Followers

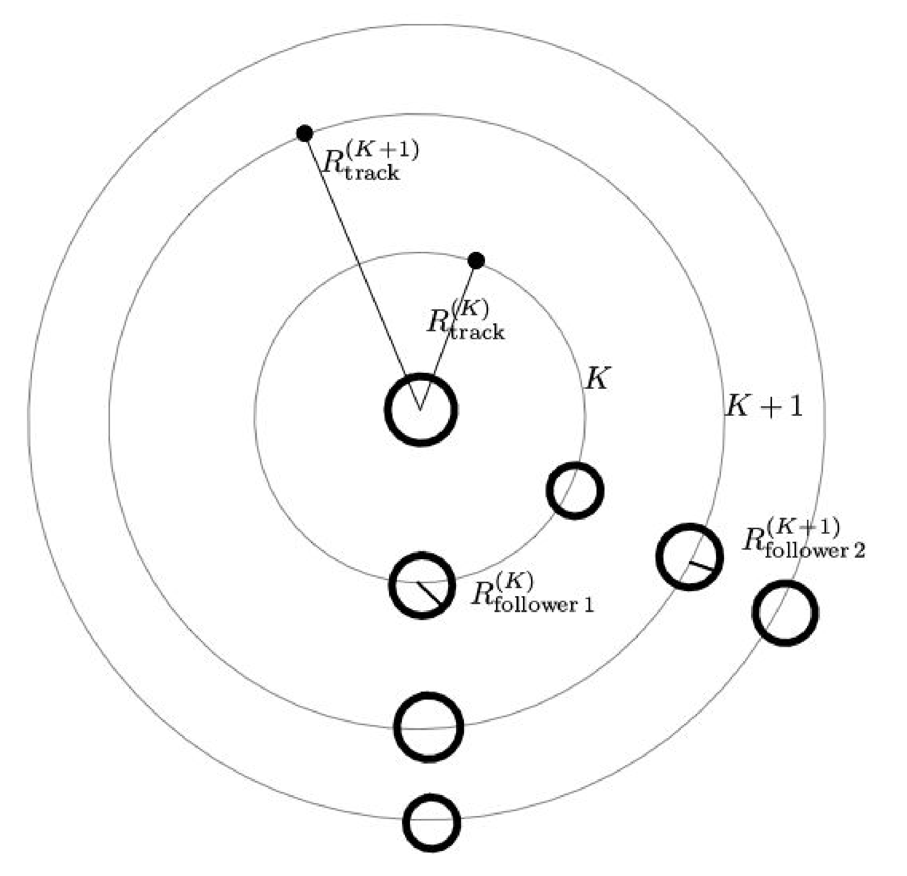
- One leader in the center
    - Potentially be virtual
    - Radius = 0
- Followers on the fixed track
    - Never touch another track
    - Followers in the same track
        - Sync the velocity
- Proof for one track
    - SAFE FOR ALL
    - Not for a specific track radius



Safe for one ---> Safe for all

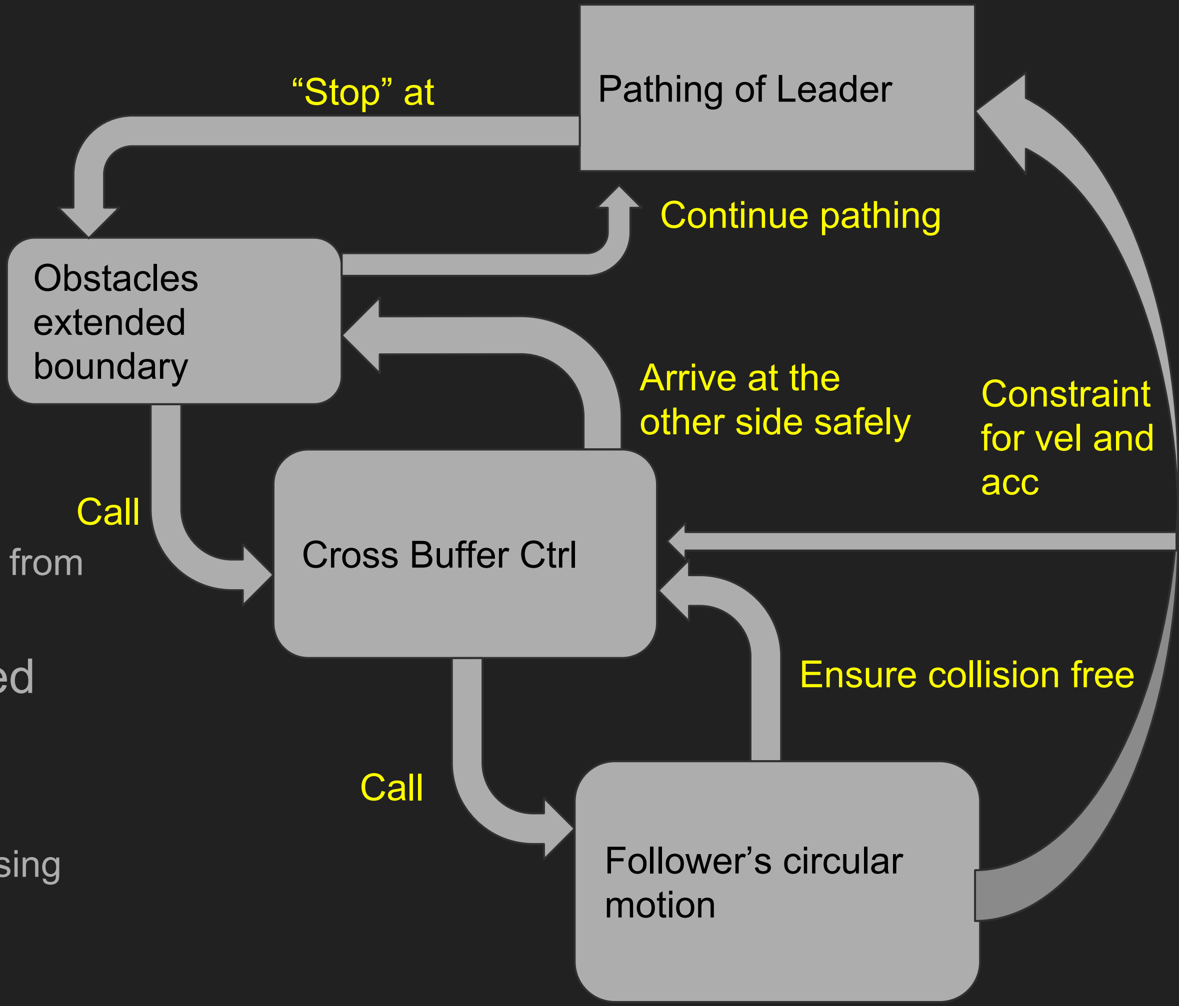
One simple invariant

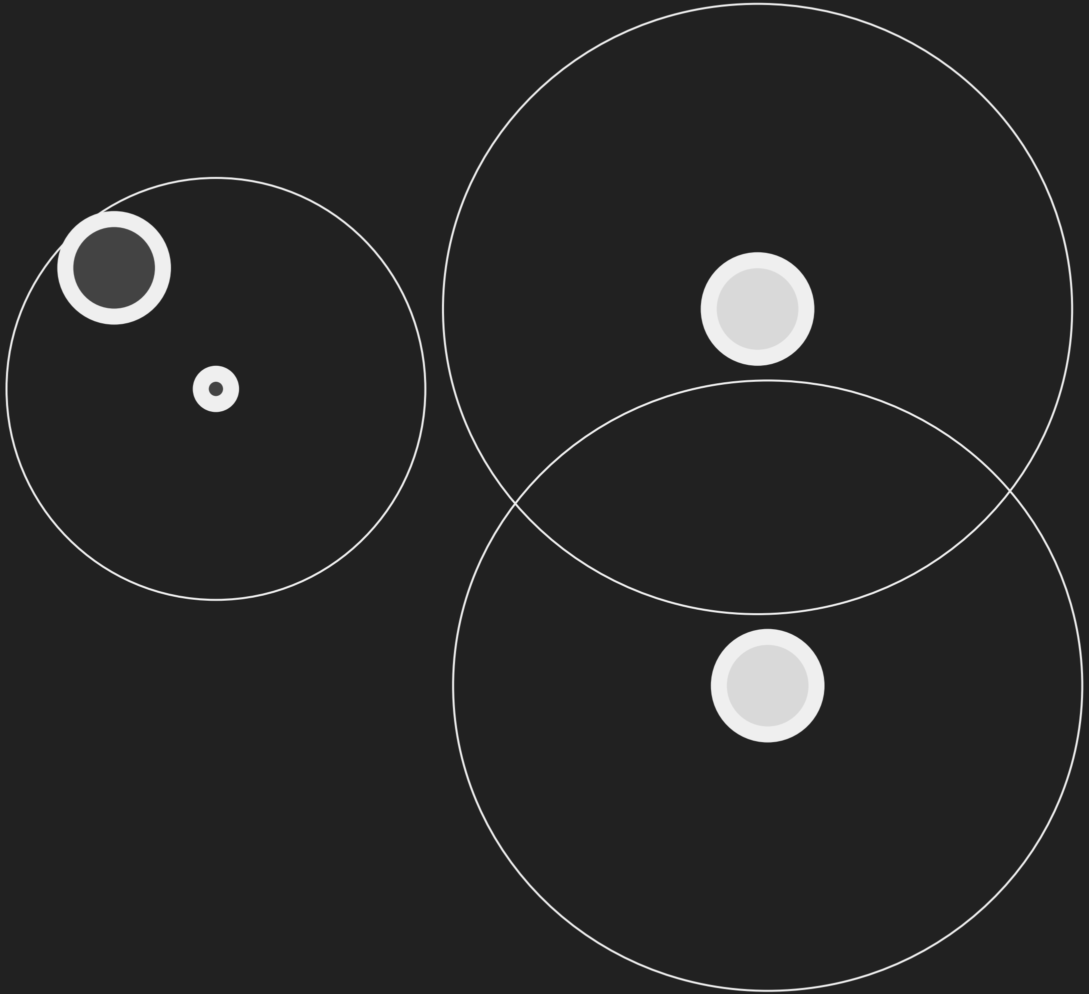
$$R_{\text{track}}^{(K)} + \max\left(\bigcup_{i \in F^{(K)}} \{R_{\text{follower } i}^{(K)}\}\right) \leqslant R_{\text{track}}^{(K+1)} - \max\left(\bigcup_{i \in F^{(K+1)}} \{R_{\text{follower } i}^{(K+1)}\}\right)$$

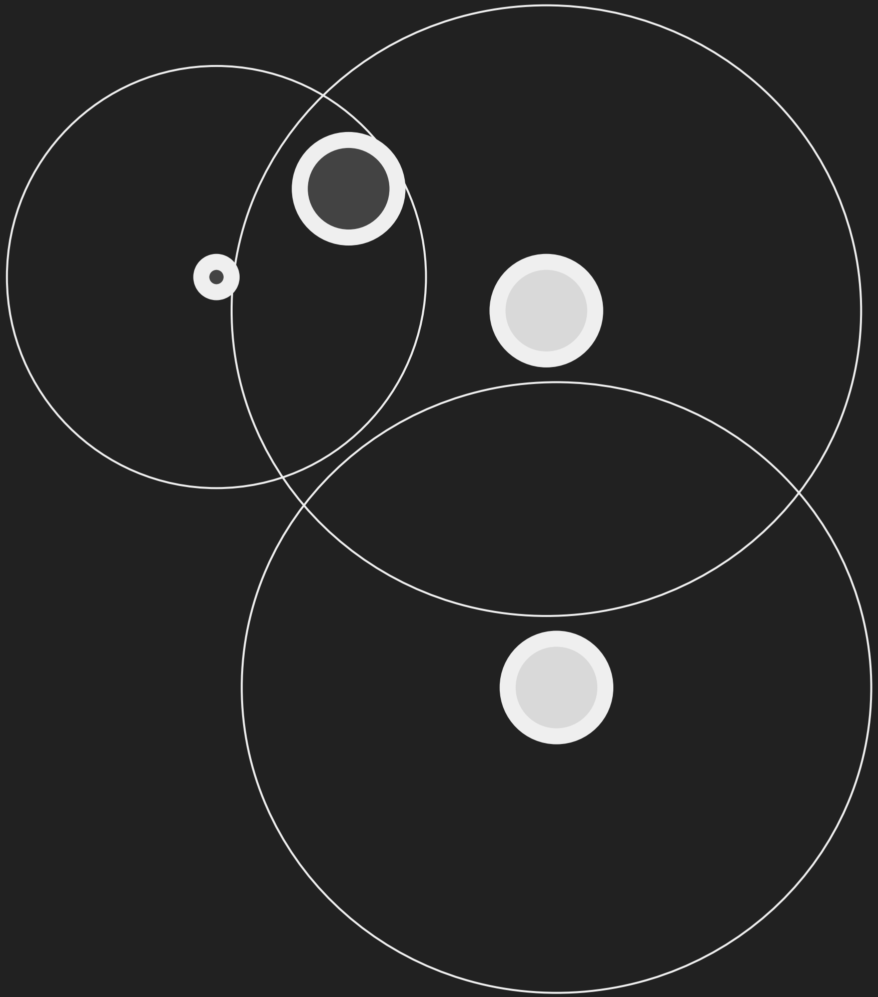


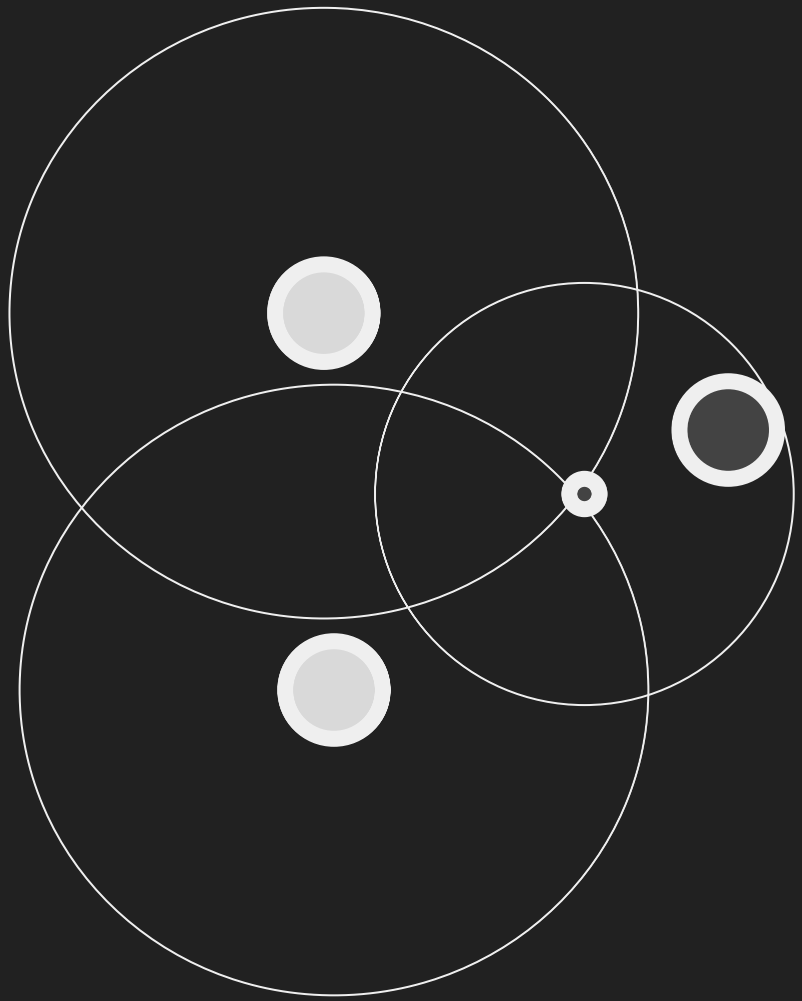
Framework Outline

- Follower's circular motion
  - Contracts
    - Control
    - Safety
    - Requirements
- Obstacles modeling
  - Keep Certain distance away from the obstacle would be safe
- Cross Safe Buffer(Extended Boundary)
  - Cross the buffer
    - Different types of crossing control
- Pathing Algorithm



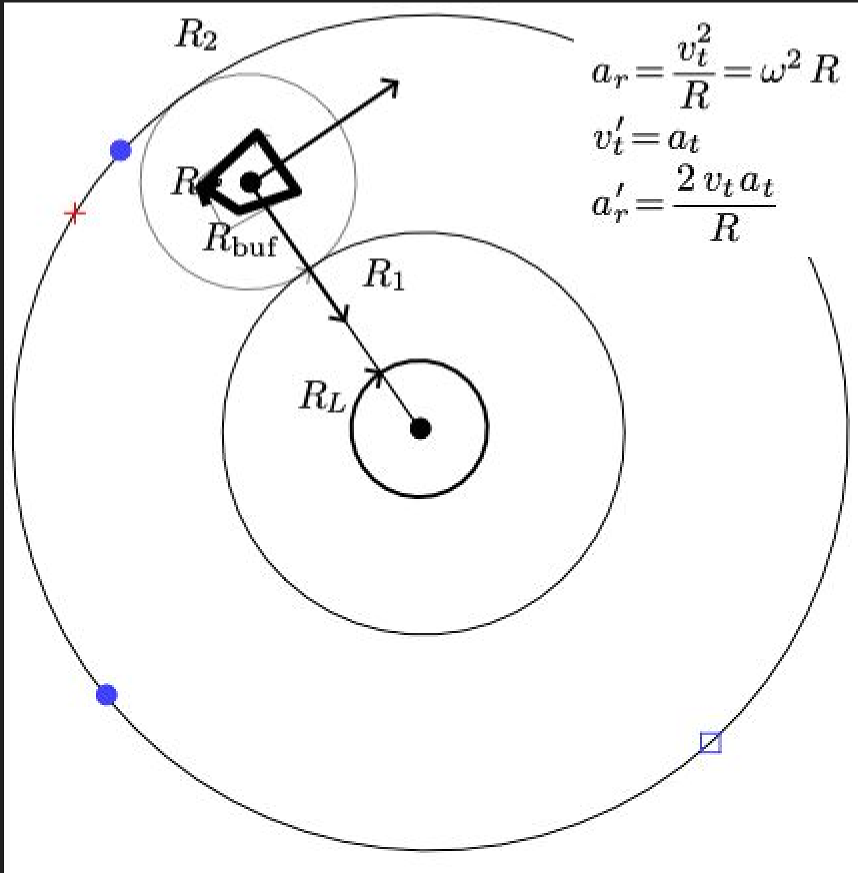






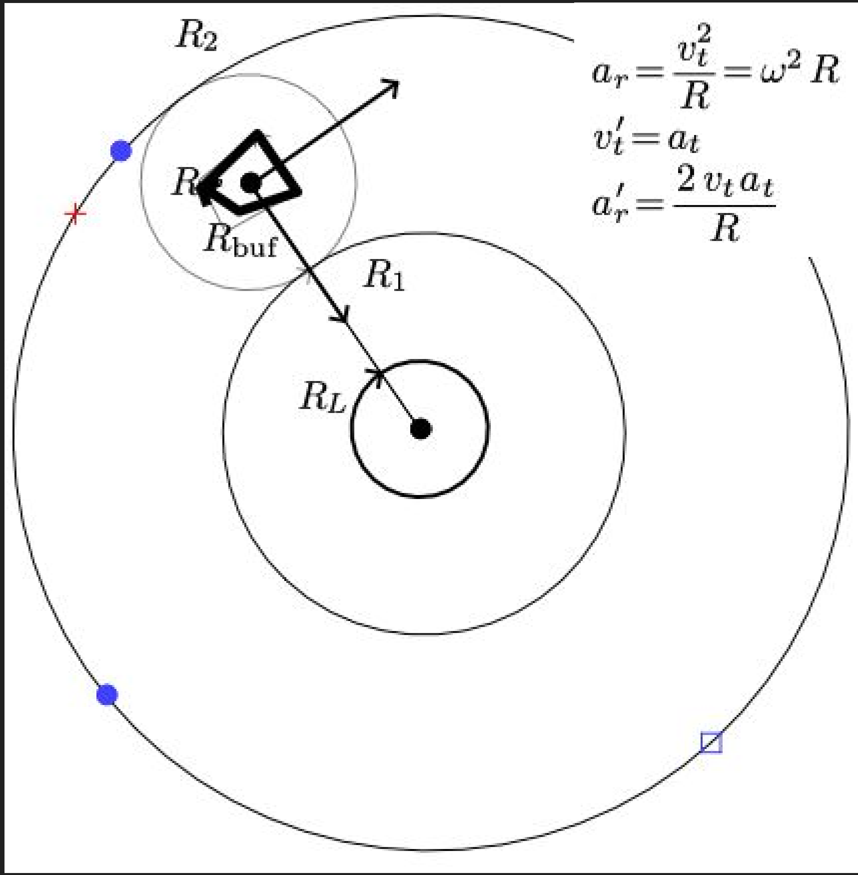
Followers’ Circular Motion

- Formal Contract
  - Control
    - API
      - Perform angular acceleration around leader
  - Safety (Ensures)
    - never go out of the fixed track
      - When not asked
    - if multiple followers on same track
      - they never collide
        - sync is one option
      - Indication flag of whether on the same half circle



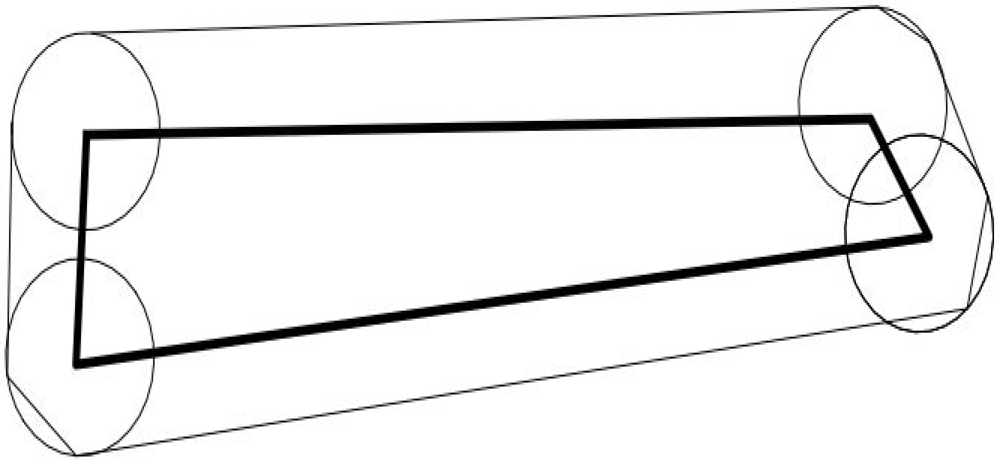
Followers' Circular Motion

- Formal Contract
  - Control
  - Safety (Ensures)
  - Requirement
    - Constraints on leader's speed and acceleration when moving around the leader
    - Eg.
      - Cars, Ships
      - Walking robots
      - UFO, disklike vehicles



Obstacles modeling

- Extended boundaries (Safe Buffer)
  - Depend on the longest radius of followers
  - Draw such circles at all nodes
  - Connect them using tangent lines
  - Collision free
    - Zero radius speed at boundary
  - Between any point on the boundary and any point on the obstacle at least R distance

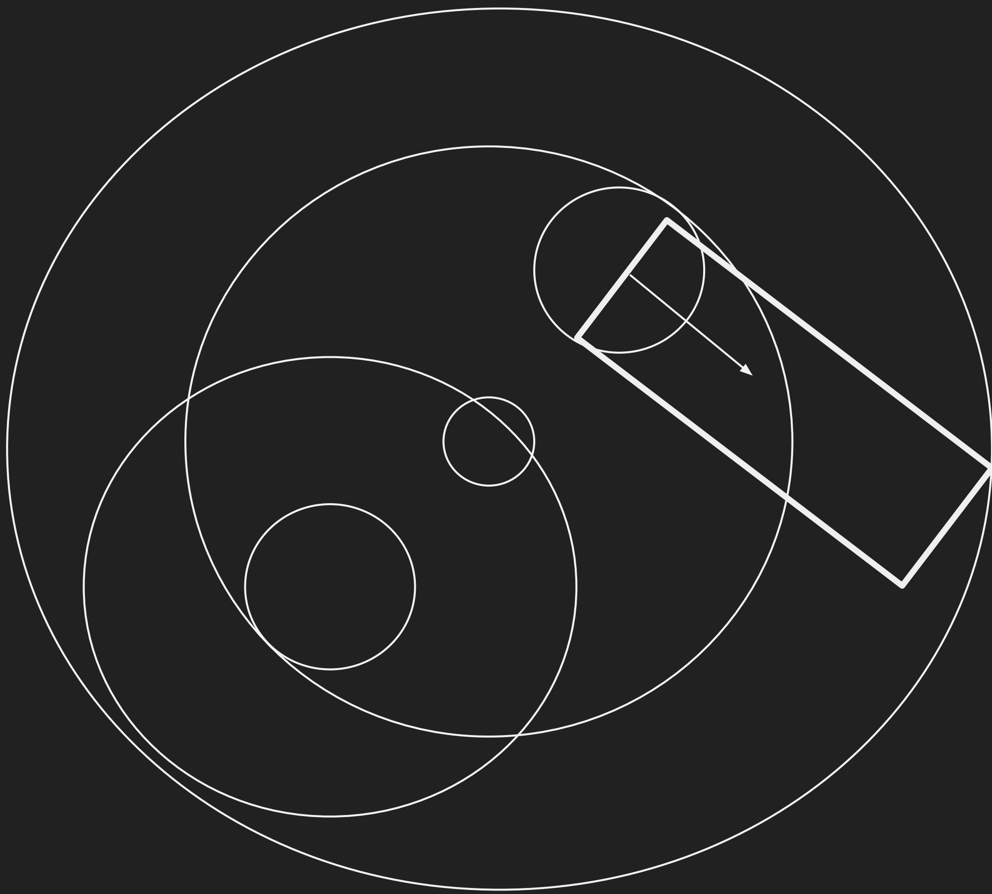


$$(x_{\text{leader}} - x_{\text{ob}})^2 + (y_{\text{leader}} - y_{\text{ob}})^2 = R^2_{\text{boudary}}$$
$$\rightarrow v_x \times (x_{\text{ob}} - x_{\text{leader}}) + v_y \times (y_{\text{ob}} - y_{\text{leader}}) \leqslant 0$$

$$\text{Dis}_{\text{ex}} \geqslant R^{(N)}_{\text{track}} + \max\left(\bigcup\nolimits_{i\in F^{(N)}} \{R^{(N)}_{\text{follower } i}\}\right)$$

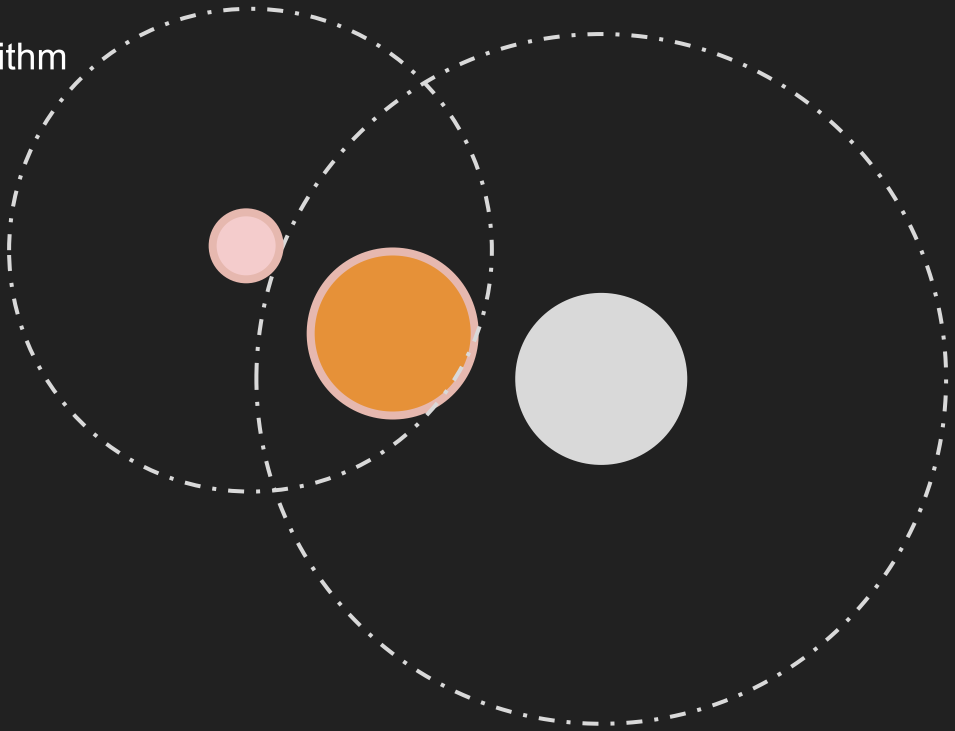
Obstacles modeling

- Other ways to define Extended boundaries (Safe Buffer)
  - Still depend on the longest radius of followers
  - Draw larger circles
    - Follower have detect range
  - Collision free outside
    - Not necessarily Zero radius speed at boundary (efficiency)
  - Cons
    - Circular motion while moving
  - Pros
    - No need for everyone to detect all the time



Cross Safe buffer

Proved Basic algorithm



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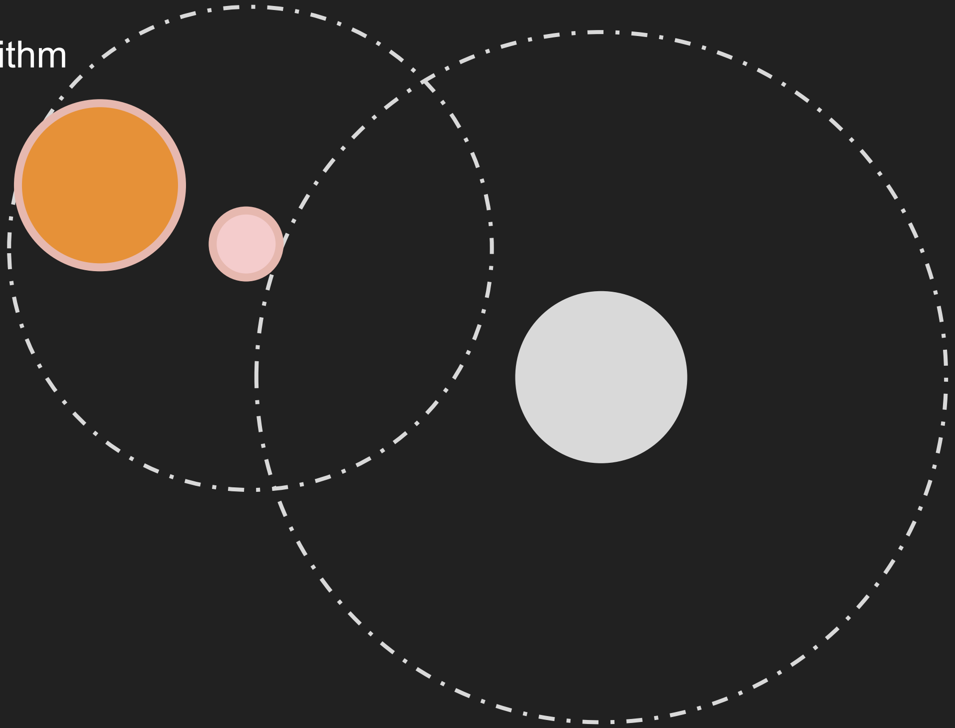

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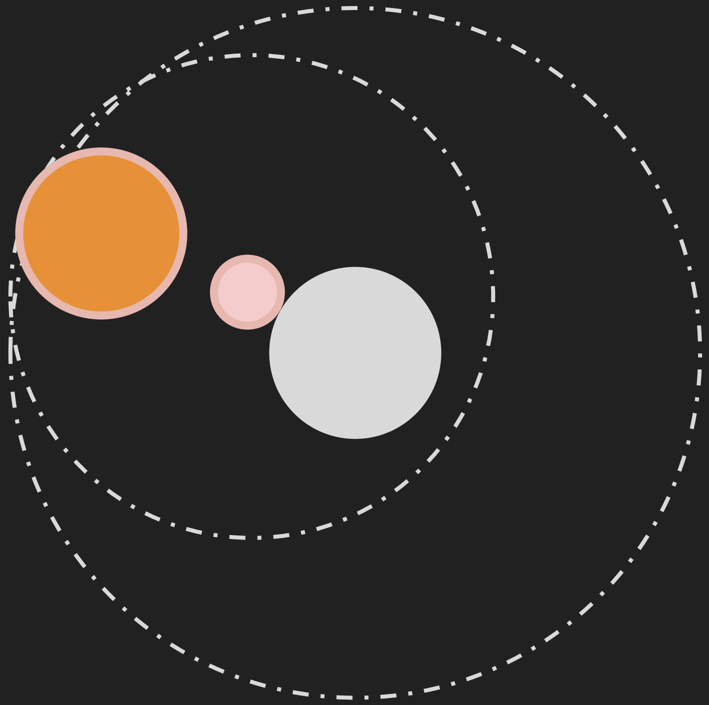

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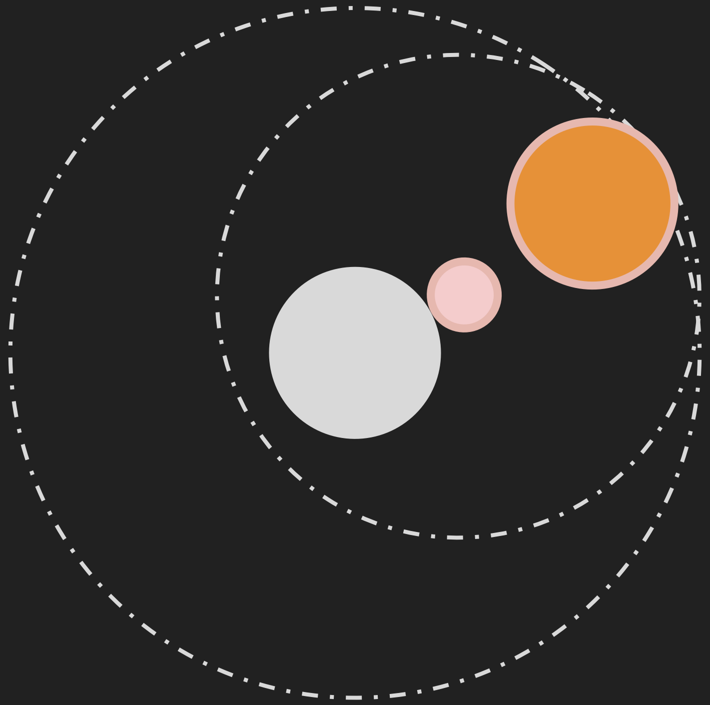

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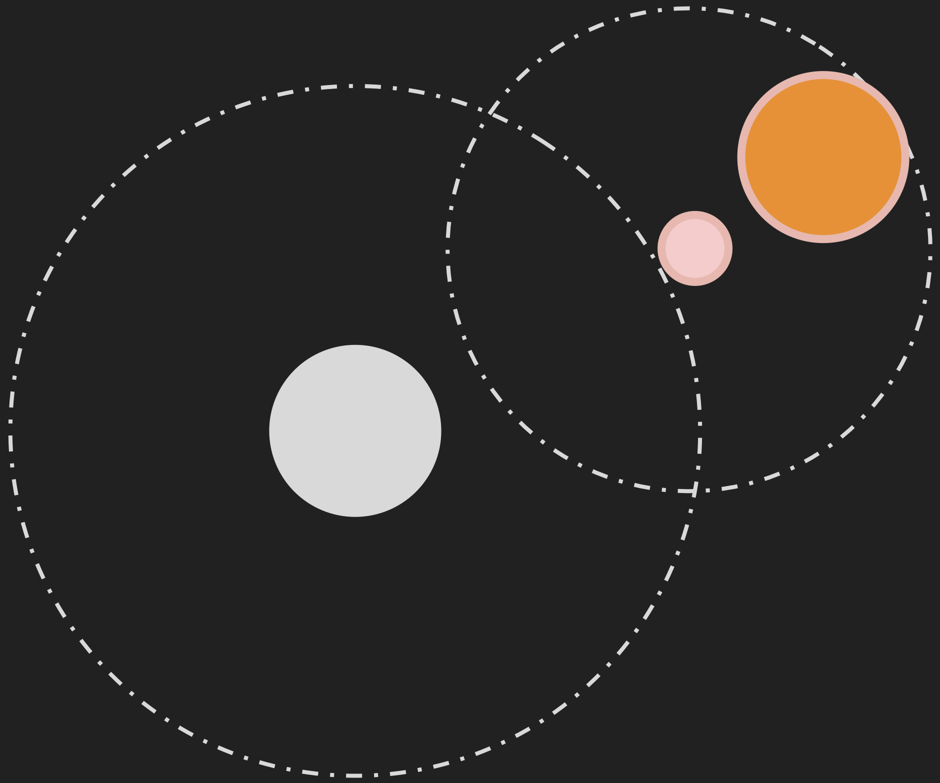
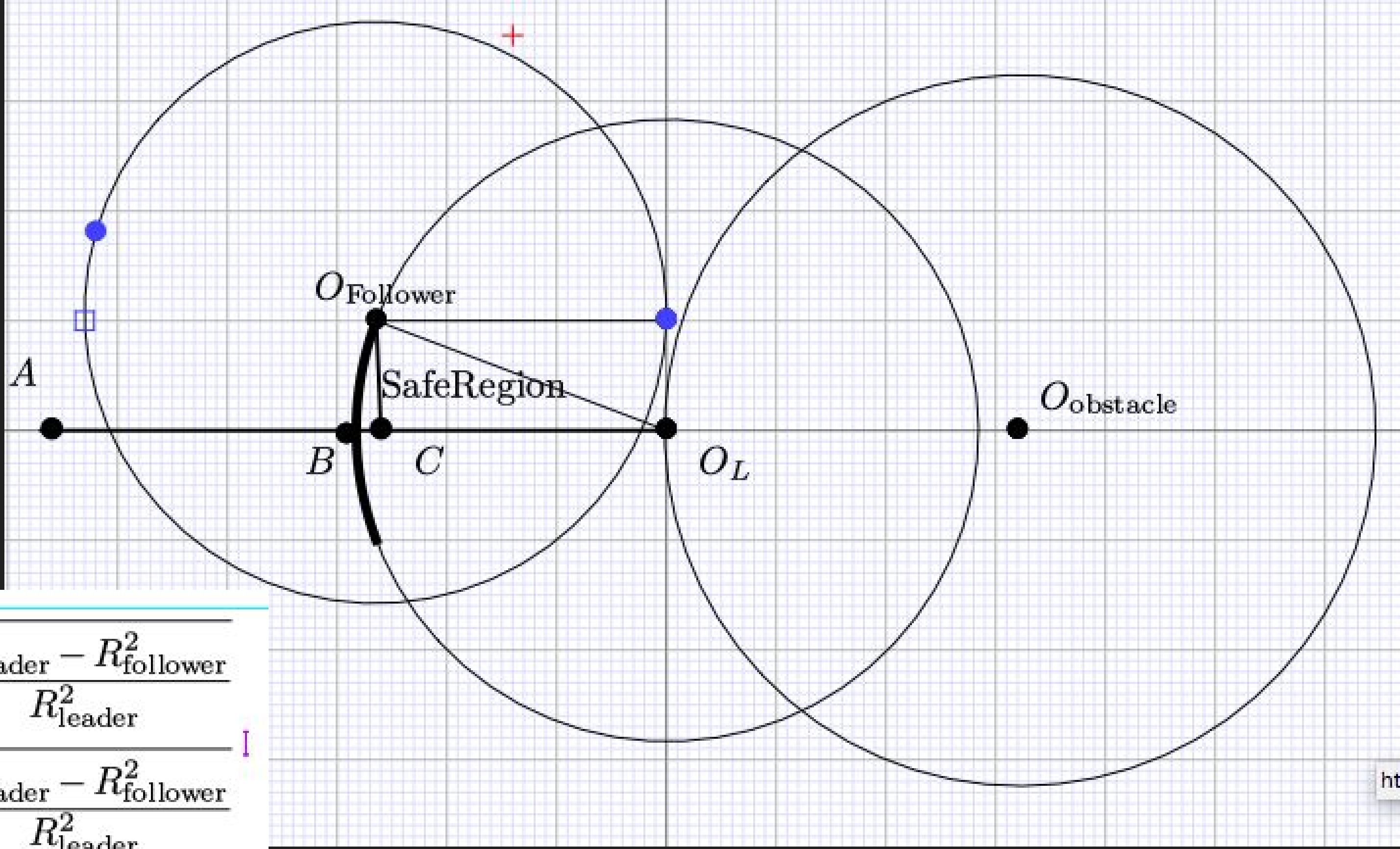


Cross Safe buffer

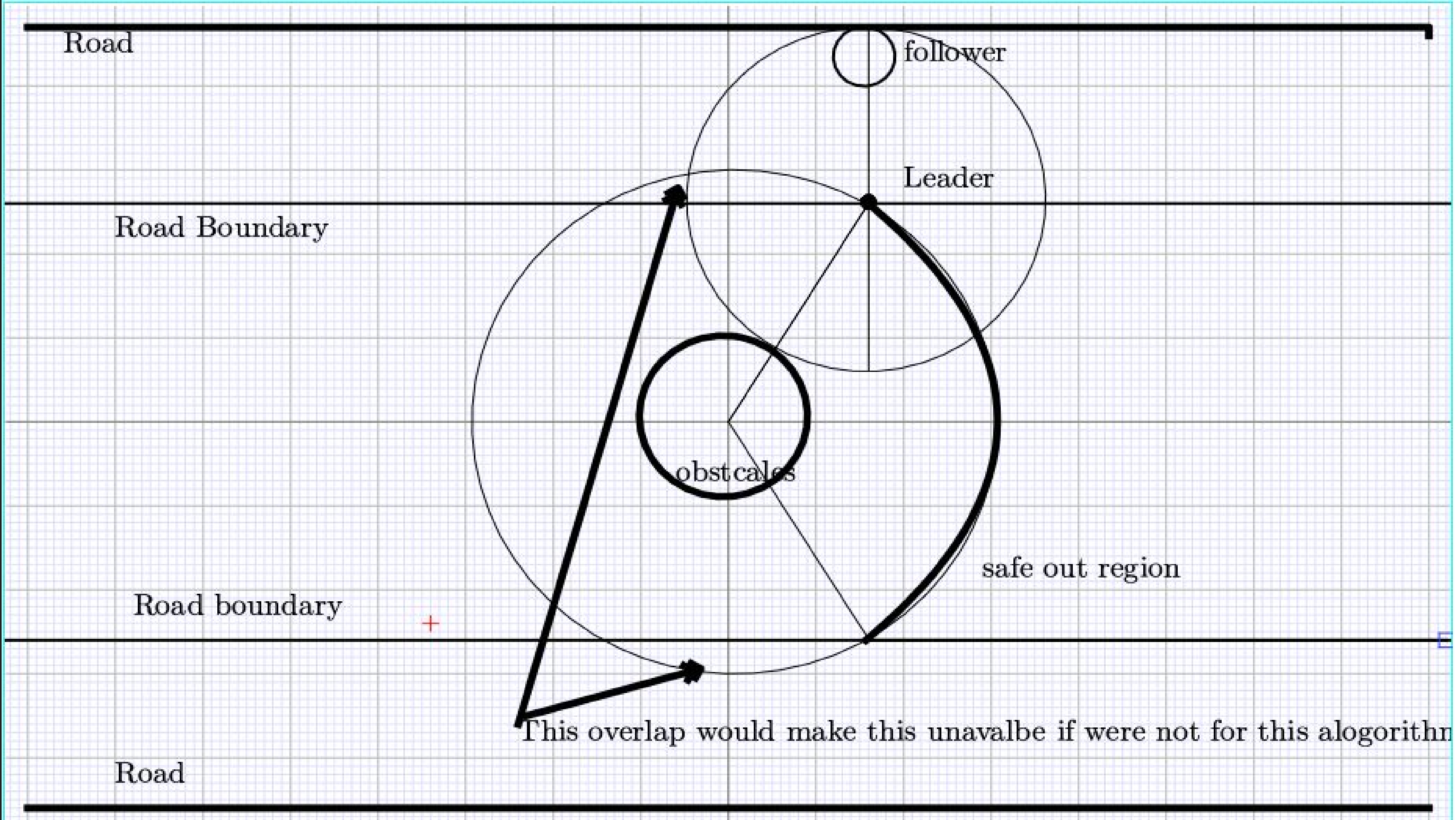
- Leader’s movement never happens together with Followers’ circular motion
- Compatible with most circular motion strategies

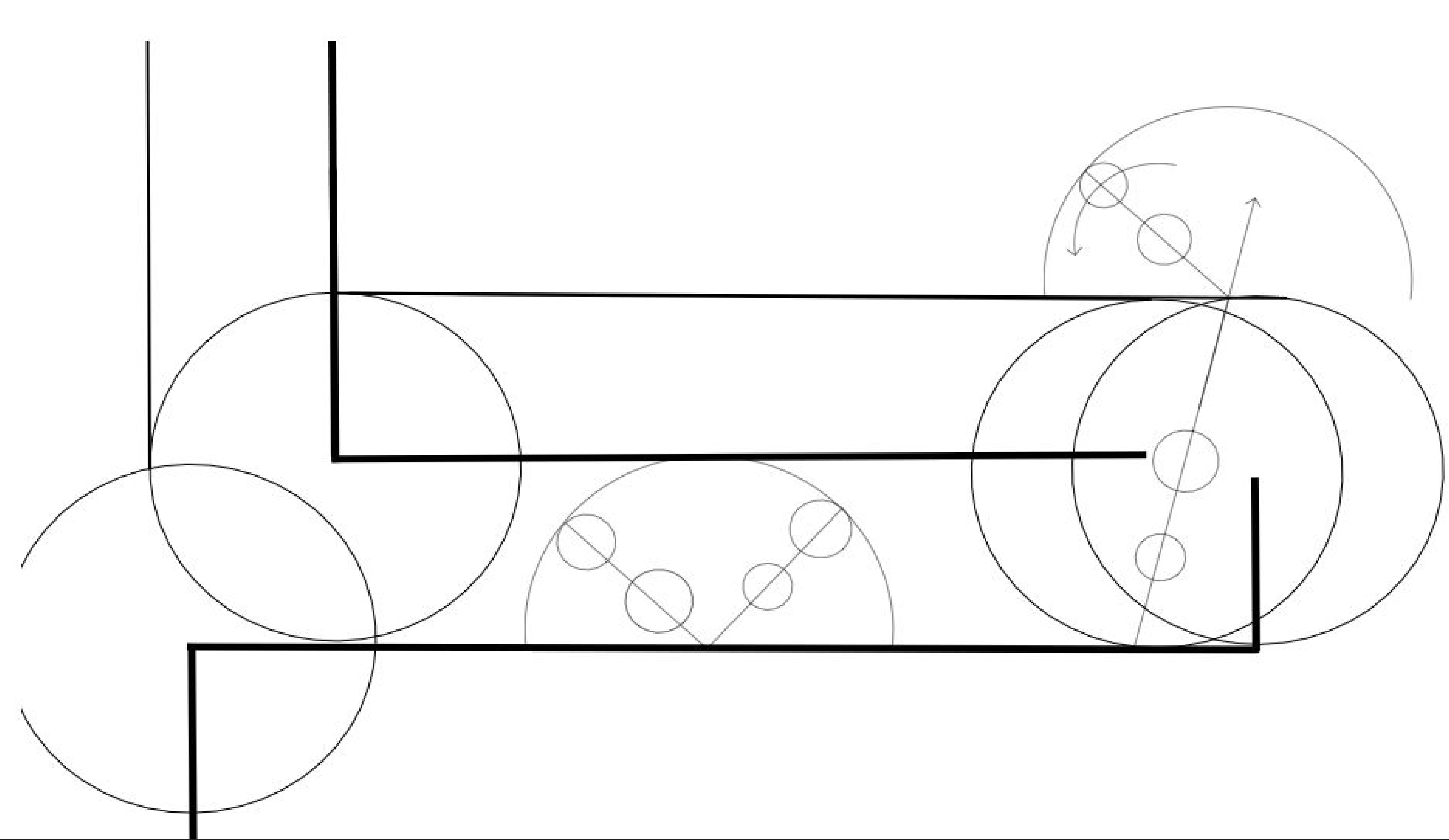
Safety



$$\text{safe rad} \leqslant 3.14 + \sqrt{\frac{R_{\text{leader}}^2 - R_{\text{follower}}^2}{R_{\text{leader}}^2}}$$

$$\text{safe rad} \geqslant 3.15 - \sqrt{\frac{R_{\text{leader}}^2 - R_{\text{follower}}^2}{R_{\text{leader}}^2}}$$


Road
follower
Leader
Road Boundary
obstcales
safe out region
Road boundary
This overlap would make this unavalbe if were not for this alogorithn
Road



Pathing

- Satisfy the constraint given by safe buffer
- Satisfy the constraint given by circular motion layer
  - No need for circular motion in pathing due to our extended boundary
- In my case, It is just a simple one moving boids avoiding static obstacles
- We could implement A star on top of this, and mark points on extended boundary are accessible to each other.
  - Call cross buffer algorithm to access
- This could be done manually as well
  - Driver needs to follow the constraint

Split and Merge

$$\mathrm{Dis}_{\mathrm{ex}} \geqslant R^{(N)}_{\mathrm{track}} + \max\left(\bigcup\nolimits_{i\in F^{(N)}} \left\{R^{(N)}_{\mathrm{follower}\,i}\right\}\right)$$

$$R^{(K)}_{\mathrm{track}} + \max\left(\bigcup_{i\in F^{(K)}} \left\{R^{(K)}_{\mathrm{follower}\,i}\right\}\right) \leqslant R^{(K+1)}_{\mathrm{track}} - \max\left(\bigcup_{i\in F^{(K+1)}} \left\{R^{(K+1)}_{\mathrm{follower}\,i}\right\}\right)$$

- Split
  - Let the leader stop and outermost followers stop
  - the splitted followers become obstacles
    - Just satisfy our extended boundary requirement
- Merge
  - Stop at the extended boundary of the to-be-merged follower.
  - Then the follower is just at its track radius away from the leader
  - Then we can remove that extended boundary generated by the follower
  - Send control to the follower to make it follow
  - Need to update all other extended boundary size accordingly.
    - After merging , leader should not inside any other safe buffer

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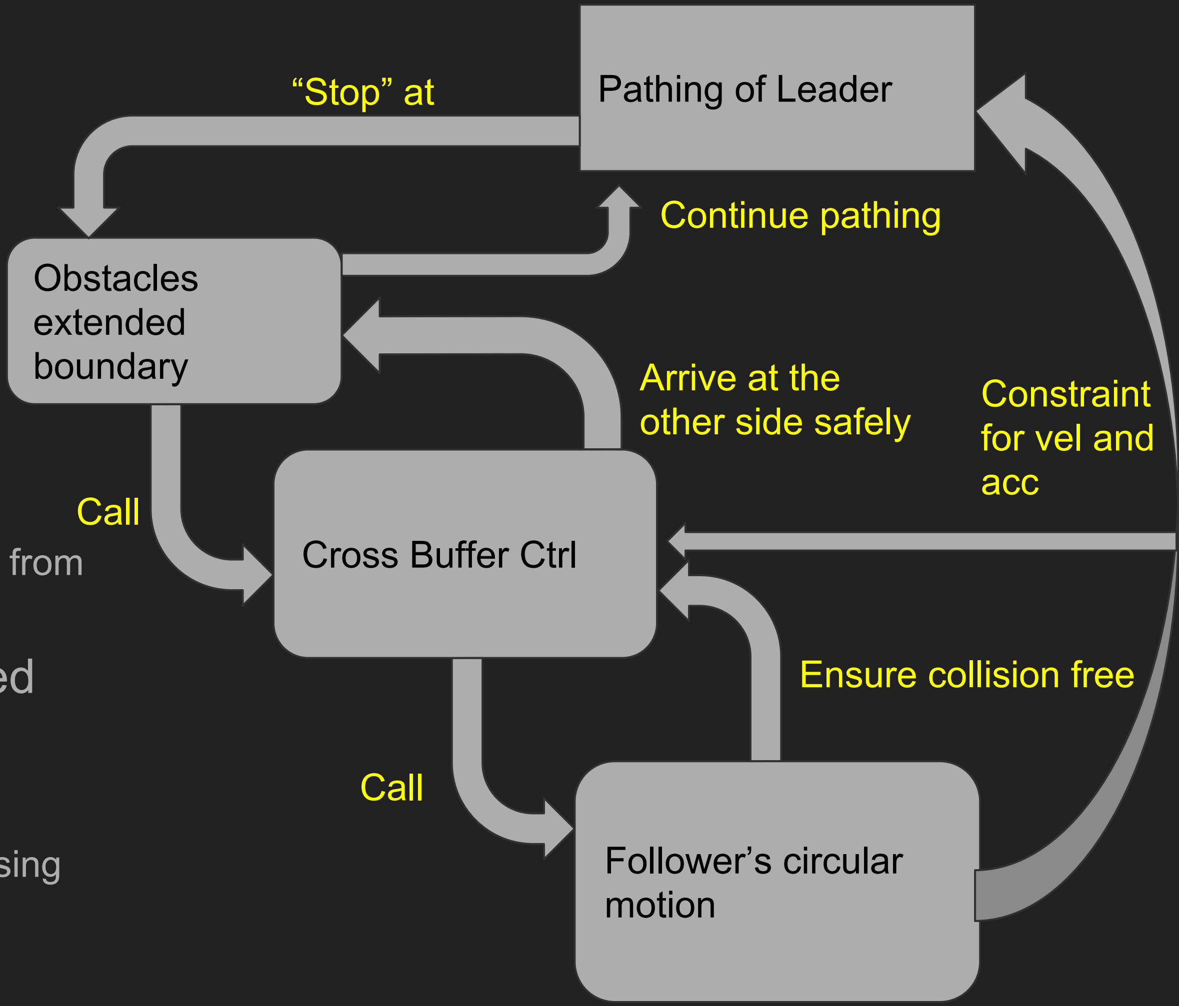

The End

Special credit to my TA Nathan, I got this project idea when discussing with him 4 days before deadline, and decide to change to this, i think is cool, idea

Thanks for everyone listening , any question?

ZIHAN ZHOU
zihanz@andrew.cmu.edu
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