Flock on the March

A Meta-Proof of a Meta-Model

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What am I modeling?

- **Boids**
  - Infinite many
  - Same destination
  - Not a transportation protocol

- **Obstacles**
  - Infinite many
  - Arbitrary shape
  - Arbitrary size
  - Pass some
    - Distant enough
Project Highlights

- **Without Distributed DL!**
  - This model is constructed in a certain way

- **Framework, Modular**
  - Control
  - Proof

- **Analogy**
  - Functions
  - Specs for functions
    - Contracts
Leader-Followers

- One leader in the center
  - Potentially be virtual
  - Radius = 0
- Followers on the fixed track
  - Never touch another track
  - Followers in the same track
    - Sync the velocity
- Proof for one track
  - SAFE FOR ALL
  - Not for a specific track radius
Safe for one ---> Safe for all

One simple invariant

\[ R_{track}^{(K)} + \max \left( \bigcup_{i \in F^{(K)}} \{ R_{follower i}^{(K)} \} \right) \leq \]

\[ R_{track}^{(K+1)} - \max \left( \bigcup_{i \in F^{(K+1)}} \{ R_{follower i}^{(K+1)} \} \right) \]
Framework Outline

- Follower’s circular motion
  - Contracts
    - Control
    - Safety
    - Requirements
- Obstacles modeling
  - Keep Certain distance away from the obstacle would be safe
- Cross Safe Buffer (Extended Boundary)
  - Cross the buffer
    - Different types of crossing control
- Pathing Algorithm
Followers’ Circular Motion

- **Formal Contract**
  - Control
    - API
      - Perform angular acceleration around leader
  - Safety (Ensures)
    - never go out of the fixed track
      - When not asked
    - if multiple followers on same track
      - they never collide
        - sync is one option
    - Indication flag of whether on the same half circle

\[
a_r = \frac{v_i^2}{R} = \omega^2 R
\]
\[
v_i' = a_t
\]
\[
a_r' = \frac{2 v_t a_t}{R}
\]
Followers’ Circular Motion

- Formal Contract
  - Control
  - Safety (Ensures)
  - Requirement
    - Constraints on leader’s speed and acceleration when moving around the leader
    - Eg.
      - Cars, Ships
      - Walking robots
      - UFO, disklike vehicles
Obstacles modeling

- Extended boundaries (Safe Buffer)
  - Depend on the longest radius of followers
  - Draw such circles at all nodes
  - Connect them using tangent lines
  - Collision free
    - Zero radius speed at boundary
  - Between any point on the boundary and any point on the obstacle at least R distance
Obstacles modeling

- Other ways to define Extended boundaries (Safe Buffer)
  - Still depend on the longest radius of followers
  - Draw larger circles
    - Follower have detect range
  - Collision free outside
    - Not necessarily Zero radius speed at boundary (efficiency)
  - Cons
    - Circular motion while moving
  - Pros
    - No need for everyone to detect all the time
Cross Safe buffer

Proved Basic algorithm
Cross Safe buffer

Proved Basic algorithm
Cross Safe buffer

Proved Basic algorithm
Cross Safe buffer

- Leader’s movement never happens together with Followers’ circular motion
- Compatible with most circular motion strategies
Safety

\[
\text{safe rad } \leq 3.14 + \sqrt{\frac{R^2_{\text{leader}} - R^2_{\text{follower}}}{R^2_{\text{leader}}}}
\]

\[
\text{safe rad } \geq 3.15 - \sqrt{\frac{R^2_{\text{leader}} - R^2_{\text{follower}}}{R^2_{\text{leader}}}}
\]
This overlap would make this unavalbe if were not for this algorithm.
Pathing

- Satisfy the constraint given by safe buffer
- Satisfy the constraint given by circular motion layer
  - No need for circular motion in pathing due to our extended boundary
- In my case, it is just a simple one moving boids avoiding static obstacles
- We could implement A star on top of this, and mark points on extended boundary are accessible to each other.
  - Call cross buffer algorithm to access
- This could be done manually as well
  - Driver needs to follow the constraint
Split and Merge

- **Split**
  - Let the leader stop and outermost followers stop
  - the splitted followers become obstacles
    - Just satisfy our extended boundary requirement

- **Merge**
  - Stop at the extended boundary of the to-be-merged follower.
  - Then the follower is just at its track radius away from the leader
  - Then we can remove that extended boundary generated by the follower
  - Send control to the follower to make it follow
  - Need to update all other extended boundary size accordingly.
    - After merging, leader should not inside any other safe buffer

Mathematical Formulas:

\[
\text{Dis}_{ex} \geq R_{track}^{(N)} + \max \left( \bigcup_{i \in F(N)} \{ R_{follower i}^{(N)} \} \right)
\]

\[
R_{track}^{(K)} + \max \left( \bigcup_{i \in F(K)} \{ R_{follower i}^{(K)} \} \right) \leq R_{track}^{(K+1)} - \max \left( \bigcup_{i \in F(K+1)} \{ R_{follower i}^{(K+1)} \} \right)
\]
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Pathing of Leader

“Stop” at

Continue pathing

Arrive at the other side safely

Constraint for vel and acc

Ensure collision free

Followed’s circular motion

Obstacles extended boundary

Cross Buffer Ctrl

Call

Call
Special credit to my TA Nathan, I got this project idea when discussing with him 4 days before deadline, and decide to change to this, I think is cool, idea.

Thanks for everyone listening, any question?

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