RollerCoaster Tycoon X

Like the original, but safer

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Who has taken a roller coaster?
Who wants to be confident that they are safe?
The safety of coasters is verified by numerical simulation
But numerical simulation is subject to error

\[
x' = -y, \ y' = x
\]

\[
x' = x
\]
Approach
We break a coaster down into track sections and prove safety individually

- Reduces complexity
- More generalizable
- Prove properties for individual sections
- Ending conditions for one section are the starting conditions for next
- Piece the different proofs back together to form a complete proof
Coasters can be modeled with straight lines and arcs
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- Straight lines: $y = mx + c$
- Arcs: $(cx-x)^2 + (cy-y)^2 = r^2$
We want to prove several properties about roller coasters

- Train goes forward
- Energy is conserved
- Train stays on the track
Models
Modeling: Basic Assumptions

- Assumptions: zero friction, unit gravity, point mass, two dimensions
Modeling: Straight Line Dynamics

- For straight lines:
  - $x' = v^*dx$
  - $y' = v^*dy$
  - $v' = g \sin \theta$
Modeling: Arc Dynamics

- For arcs (clockwise):
  - $x' = \frac{v(y-cy)}{r}$
  - $y' = -\frac{v(x-cx)}{r}$
  - $v' = \frac{(x-cx)}{r}$
Proof
Proof approach

• If ODEs are solvable, solve!
• Identify which properties would be proved through similar means
• Proving properties that were essential in proving others
  ○ Positive velocity is a powerful property!
• Useful proof rules
  ○ Differential invariants
  ○ Differential ghosts
**Proof Example: Arc motion**

1. **Stays on track and energy is conserved**
   ○ Proved with invariants

2. **Strictly positive velocity**
   ○ Proved with ghosts as velocity is decreasing

3. **Stays within the quadrant**
   ○ Proved with invariants using information about velocity
RollerCoaster Tycoon X is a safe roller coaster design tool
We modeled the Top Thrill Dragster and proved that it is safe