15-819/18-879 Hybrid Systems Analysis & Theorem Proving

Assignment 2 due by Thu 2/19/2009, hand in WEH 7120/7109

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Disclaimer: No solution will be accepted that comes without an **explanation**!

Exercise 1 First-Order Tableaux (10p)

1. Prove or disprove the following formulas using tableaux.
   a) $\big(\forall x \forall y \forall z\, (p(x,y) \land p(y,z) \to p(z,x)) \land \forall x\, p(x, f(x))\big) \to \forall x \exists y\, p(y,x)$
   b) $\big(\forall x\, (ground(x) \to cruise(x)) \land \forall x\, \forall y\, (cruise(x) \land cruise(y) \to separate(x,y)) \land \exists x\, ground(x)\big) \;\to\; \forall x\, (cruise(x) \to \exists y\, separate(x,y))$
   You can use abbreviations g, c, s for $ground, cruise, separate$, respectively.
2. Give tableau proof rules for the following operators:
   a) XOR (exclusive-or)
   b) NAND (negated and)
   c) NOR (negated or)
   d) $A?B : C$ with the semantics

$$[\![A?B : C]\!]_I = \begin{cases} [\![B]\!]_I & \text{if } [\![A]\!]_I = \mathit{true} \\ [\![C]\!]_I & \text{if } [\![A]\!]_I = \mathit{false} \end{cases}$$

Exercise 2 Tableau Calculus (5p)

Give an example showing why the application of closing substitutions in free variable tableaux to the full tableau is necessary.

Exercise 3 Propositional Tableaux (4p)

Is the following tableau rule a replacement for the implication rule?

$$\frac{A \to B}{\neg A \qquad \begin{matrix} A \\ B \end{matrix}}$$

What is the advantage of this rule? Is it a sound replacement? Is it a complete replacement for the implication rule?

Exercise 4 Sequent Calculus (4p)

1. Prove or disprove the following formulas using the sequent calculus presented in class.
   a) $C \vee \forall x\,(\neg p(x) \wedge \neg q(x))) \to ((\exists y\,(\neg q(y) \to p(y))) \to C)$

Exercise 5 Completeness of Propositional Tableaux (9p)

1. Prove formally that propositional tableaux are complete, i.e., every valid propositional formula can be proven using propositional tableaux.
2. Prove formally that propositional tableaux give a decision procedure for propositional logic.

Exercise 6 Logical Modeling (18p)

We call relation $R \subseteq D \times D$ reflexive if $\{(a,a) \;:\; a \in D\} \subseteq R$.
We call relation $R \subseteq D \times D$ irreflexive if $\{(a,a) \;:\; a \in D\} \cap R = \emptyset$.
We call relation R symmetric if $\{(a,b) \;:\; (b,a) \in R\} \subseteq R$.
We call relation R asymmetric if it is symmetric at no point, i.e., we never find $(b,a) \in R$ and $(a,b) \in R$ simultaneously.
We call relation R transitive if $\{(a,b) \;:\; (a,c) \in R, (c,b) \in R \text{ for some } c\} \subseteq R$.

1. Formalize each of those notions about relations in first-order logic.
2. Formalize the conjecture that all asymmetric relations are irreflexive.
3. Formalize the conjecture that all relations that are transitive and irreflexive are also asymmetric.
4. Prove these conjectures in KeY[1].

[1] `http://www.key-project.org/` or `http://www.key-project.org/download/releases/webstart/KeY.jnlp`