BusyBees
Safe Controllers for Multi-Agent Swarms

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Overview

- Motivation
- Prior Work
- System Model
- 1D Case
- 2D Cases
- Applications
Robotics is Hard


They used **coding and algorithms** so the drones didn't crash into each other

```java
if(goingToCrashIntoEachOther) {
  dont();
}
```

https://imgur.com/gallery/qv1gQ
Biology Makes Swarms Look Easy


https://en.wikiversity.org/wiki/Algorithm_models/Grey_Wolf_Optimizer
Goal

▸ What does it mean for a swarm to be “safe”?
▸ How do we design controllers for safe swarms?
▸ Applying kinematic principles to swarm controller design
Prior work

- Probabilistic models of swarms
  - Point masses with holonomic dynamics
  - Vector fields direct agents towards a clustering point
  - Good for describing large-scale dynamics
  - Poor at ensuring safety and collision-free behavior

Prior work

- **Barrier Certificates**
  - Define safe set bounded by some barrier function
  - Correctly defined barrier function $\Rightarrow$ always remain in safe set!
  - Provably safe collision-free controllers for n-agent swarms
  - Only physically close agents need to worry about collisions
  - Agents collaboratively brake and accelerate to avoid collision
  - Approximated differential dynamics as holonomic, not solid proof
Model design

- n-agent system of differential drive agents
  - Maximum braking and acceleration [-B,A]
  - Non-negative velocity and maximum velocity
  - Minimum turning radius

- Time-triggered controller
  - All agents make coordinated decisions

Model design

- Two safety constraints
  - Minimum distance - Can’t get too close to collide
  - Maximum distance - Can’t move too far apart or swarm disperses
- Maximum distance constraint depends on swarm structure
Model Design

- Continuous Dynamics

\[
\begin{align*}
{x}' &= v \cdot d_x, \\
{y}' &= v \cdot d_y, \\
{v}' &= a, \\
{d_x}' &= -v \cdot d_y / r, \\
{d_y}' &= v \cdot d_x / r, \\
t' &= 1
\end{align*}
\]

& (v \geq 0 \land v \leq v_{\text{max}} \land t \leq T)}

- Infinity norm rather than Euclidean for distance constraints

\[
\begin{align*}
\max(\abs{x_i - x_j}, \abs{y_i - y_j}) &\geq r_{\text{min}} \\
\max(\abs{x_i - x_j}, \abs{y_i - y_j}) &\leq r_{\text{max}}
\end{align*}
\]

For safety of agents i and j, i != j
2-Agent Train on a Line

- Two agents on a line, want to satisfy both safety constraints
- Necessity of velocity constraints
- Agents do not collaborate in acceleration decisions
- ODE needs to be only for 1D case

\[ \begin{align*}
{x_F}' &= v_F, \quad x_L' = v_L, \quad v_F' = a_F, \quad v_L' = a_L, \quad t' = 1 \\
& & \& (v_F \geq 0 \land v_F \leq v_{\text{max}} \land v_L \geq 0 \land v_L \leq v_{\text{max}} \land t \leq T)}\]
2-Agent Train on a Line

- System Invariants

\[
(x_L - x_F) + \frac{(v_L - v_F) v_L}{B} - \frac{(v_L - v_F)^2}{2B} \geq r_{\text{min}}
\]

\[
(x_L - x_F) + \frac{(v_L - v_F)(v_{\text{max}} - v_L)}{A} + \frac{(v_L - v_F)^2}{2A} \leq r_{\text{max}}
\]
2-Agent Train on a Line

- Control Decisions for Follower

\[
\begin{align*}
((x_L + v_LT + \frac{-BT^2}{2}) - (x_F + v_FT + \frac{a_FT^2}{2})) \\
+ \frac{(v_L - BT) - (v_F + a_FT))(v_L - BT)}{B} - \frac{(v_L - BT) - (v_F + a_FT))^2}{2B} \geq r_{\text{min}}
\end{align*}
\]

\[
\begin{align*}
((x_L + v_LT + \frac{AT^2}{2}) - (x_F + v_FT + \frac{a_FT^2}{2})) \\
+ \frac{(v_L + AT) - (v_F + a_FT))(v_{\text{max}} - (v_L + AT))}{A} \\
+ \frac{(v_L + AT) - (v_F + a_FT))^2}{2A} \leq r_{\text{max}}
\end{align*}
\]
2-Agent Train on a Line

- Proof of Safety
  - Straightforward due to solvable ODE’s
  - Follower control decisions are derived from kinematics
  - Concern of vacuosity of control decisions
n-Agent Train on a Line

- Note the atomic nature of the 2-Agent controller
  - Leader agent does not base control decisions on the state of the Follower
  - Agent $i$ makes control decisions based upon state of agent $i-1$
  - $n$-agent system is now $n-1$ 2-Agent system
- We can’t model and prove an $n$-agent system with $dL$ and KeYmaera X
- QdL and inductive arguments must suffice for now
n-Agent Train on a Line

- Convert the 2-Agent controller to QdL

\[ ctrl \equiv \forall i : Ca(i) := \ast ; (?a(i) \leq A \land a(i) \geq B) \land closeSafetyConstraint(i - 1, i) \land farSafetyConstraint(i - 1, i) \]

\[ evol \equiv t := 0 ; \forall i : Cx(i)' = v(i) , v(i)' = a(i) , t' = 1 \& v(i) \geq 0 \land v(i) \leq v_{\text{max}} \land t \leq T \]
n-Agent Train on a Line

- Proving safety of n-Agent System
  - Need to worry about transitive safety of the system
  - Gödel’s Generalization Rule helps the proof become modular
  - Proof of 2-Agent system allows for application to the n-Agent case
  - Change control or dynamics in 2-Agent, generalizes to n-Agent

\[
\phi \rightarrow \psi \\
[\alpha] \phi \rightarrow [\alpha] \psi
\]
Agents on a Plane

- Moving from 1D to 2D with rotational dynamics is hard
  - Modified 1D controls should work for holonomic agents
  - Circular dynamics makes even the 2-agent case extremely challenging
  - Maximum distance safety constraint becomes the source of challenges
- Currently modeled system has all agents having the same controls
- Let’s look at the challenges and insights
Agents on a Plane

- 2-Agent train in 2D
  - Control system must have coordinated actions between leader and follower
  - Large minimum turning radius forces collaborative actions
  - Velocity-dependent minimum turning radius may bring further insights
Agents on a Plane

- n-Agent Train in 2D
  - Can no longer atomically consider pairs of adjacent agents
  - Train crossing on itself can result in collisions, no longer modular
  - Can constrict motion to only one direction, prevent large changes in orientation
Agents on a Plane

- n-Agent Heterogeneous Cluster
  - An advanced n-Agent train controller will likely be applicable
  - Quadratic increase in number of safety constraints
  - Need to identify when agents need to collaborate in control decisions
Agents on a Plane

- n-Agent Homogeneous Cluster
  - Center of Mass dynamics are very similar to dynamics of each agent
  - Differential invariants applied to agents can be applied to COM as well
  - COM constrained by dynamics of fastest-moving agent in swarm

\[ M' = \frac{1}{\sum_{i=1}^{n} m_i} \sum_{i=1}^{n} m_i x_i' \]
Applications

- **n-Agent Heterogeneous Cluster**
  - Agents moving within constrained factory environment
  - Use immobile “dummy” leader agent to model walls of factory

- **n-Agent Train in 1D**
  - Biomedical applications
  - Drug delivery robots in arteries
  - Robotic catheters for clearing blood clots

https://money.cnn.com/2014/05/22/technology/amazon-robots/index.html
Summary

- Safety of 2-Agents moving along line
- Modularity of 2-Agent Train control in 1D extends to n-Agent
- Analysis of challenges in the 2D case, future work