Focusing for $d\mathcal{L}$

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Develop a focused version of Differential Dynamic Logic (dL) [3], with the intent that it serve as a basis for future work on the proof theory of dL.
What is focusing?

Focused systems of proof, first described by Andreoli [1], restrict what proofs can be constructed. Each focused proof corresponds to a set of unfocused proofs.

Two major restrictions:

- Apply “invertible” proof rules when possible.
- When no invertible rules can be applied, “focus” on a formula and apply non-invertible rules to it until no longer possible.
dL, or Differential Dynamic Logic, is the system of logic we use to model the behaviour of hybrid systems and to prove properties of those models.
Followed (at a high level) the approach of Simmons [4]:

- Split the connectives of the logic into *synchronous* and *asynchronous* based on their behaviour when broken down by proof rules.
- Modify the sequent calculus to distinguish the two phases of proof construction.
- Prove logical properties of the resulting system (cut elimination, identity expansion)
- Derive soundness and completeness results from those properties
Results

- A sound (but not complete) focused system for $d\mathcal{L}$.
- Completeness fails (for this particular system) due to iteration $[\alpha^*]$.
- Iteration breaks both cut elimination and identity expansion in this focused setting for separate reasons.
What goes wrong with iteration?

Two separate issues:

- Cut elimination fails (or at least is difficult to prove) because of the *global rules* that break down iteration — the rules for loop invariants and variants.
- (The proof of) identity expansion fails because the rules for breaking down iterations do not reduce the formula to one that is structurally simpler.
Future Work

• Fix the issues with iteration to arrive at a sound and complete focused system.
• Investigate how such a system may be of use for normalizing proofs in a more general sense.


