Modeling the Dynamics of a Smart Dental Drill

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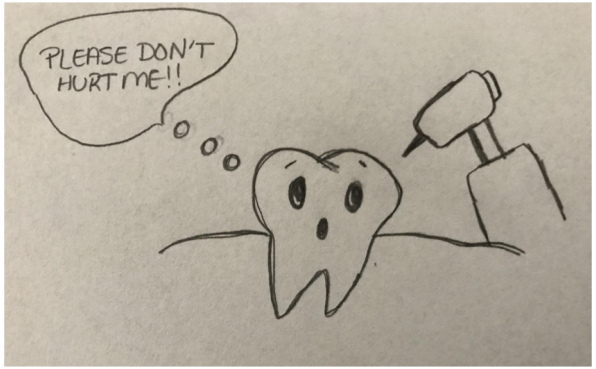
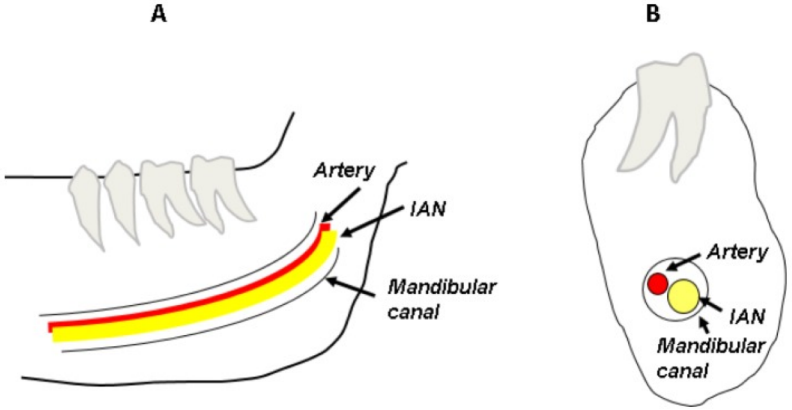
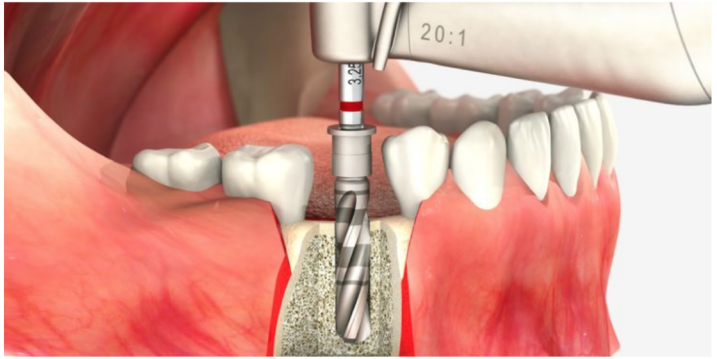
15-424



What is a Smart Dental Drill?

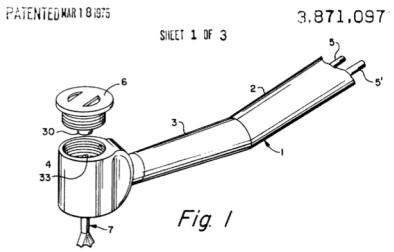
- A drill used during dental implant surgeries, which provides updates on the location of the drill in reference to the neurovascular bundle
- Provides an opportunity to drill deeply enough for the implant to hold while still ensuring safety

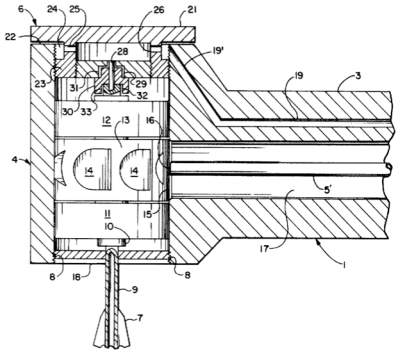




Why Should We Model Them?

- 1 million implants are inserted annually and that rate is expected to increase by 12-15% for the next several years
- We should use the new availability of information to guarantee that the procedure will always be safe with this new drill
- May lead to more successful dental implants in the future





Angular Motion Model

- Event-triggered Model
- Considers how angular motion of the drill affects the drill's downward movement
- Friction is not considered
- The user adjusts torque and weight on bit (WOB) to be any arbitrary values until it wants to stop at which point both torque and WOB become 0

$$ROP = -a1 + a2 * WOB + a3 * \Omega$$

$$\Omega' = \frac{1}{Jr} * (-c * \Omega + u)$$

Our Control Variables and our ODEs

```
 {
   wob := *; ?(wob > 0 & ((a2 * Jr * omega) / c) < d - pos); ++ wob := 0;
 }
 {
   u := *; ?(u > 0 & ((a2 * Jr * omega) / c) < d - pos); ++ u := 0;
 }

 {
     pos' = a1 * wob + a2 * omega, omega' = (1/ Jr) * (-c * omega + u) & omega >= 0 &
                                            ((a2 * omega * Jr) / c) <= d - pos
 }
olution domain and event trigger */
 ++
 {
     pos' = a1 * wob + a2 * omega, omega' = (1/ Jr) * (-c * omega + u)  & omega >= 0 &
                                            ((a2 * omega * Jr) / c) >= d - pos
 }
```

Safety Condition Explanation

- Ensures that the drill still hasn't hit the neurovascular bundle and it can slow down when no more weight on bit or torque is added to not hit the bundle
- The change in angular velocity is a decaying function

$$\Omega' = \frac{1}{Jr} * (-c * \Omega + u)$$

$$\Omega(t) = \Omega_0 e^{\frac{-c*t}{Jr}}$$

$$(\frac{(a2 * Jr * \Omega)}{c}) \leq d - pos)$$

Friction Model

- Very similar to the angular motion model
- Doesn't have stick-slip motion
- Added friction which affects the angular velocity of the drill and thus the downward movement
- Distinguished between static and kinetic friction

$$T_{f_b} = f_b * sign(x_3), f_b = W_{ob} R_b [u_{cb} + (u_{sb} - u_{cb}) exp(-y_b / v_p |x_3|)]$$

Setting Friction Within Our Model

```
{
    ?(omega > 0 | (1 / Jr) * u > sfriction * wob * rob);
    friction := kfriction * wob * rob; ++
    ?(omega = 0 & (1 / Jr) * u <= sfriction * wob * rob);
    friction := (1 / Jr) * u;
}
```

Time-Triggered Model

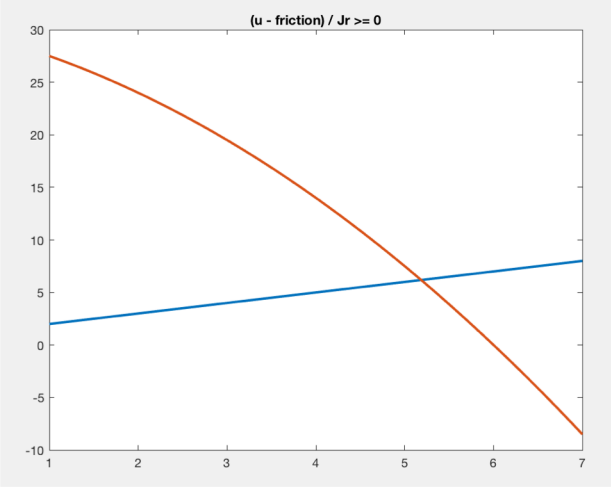
- A more realistic model that only allows for access to information from the drill within intervals of T seconds
- Upper bound on Ω and pos for simplification of proof
- Want to ensure that the ending position and ending angular velocity of the drill will still be safe

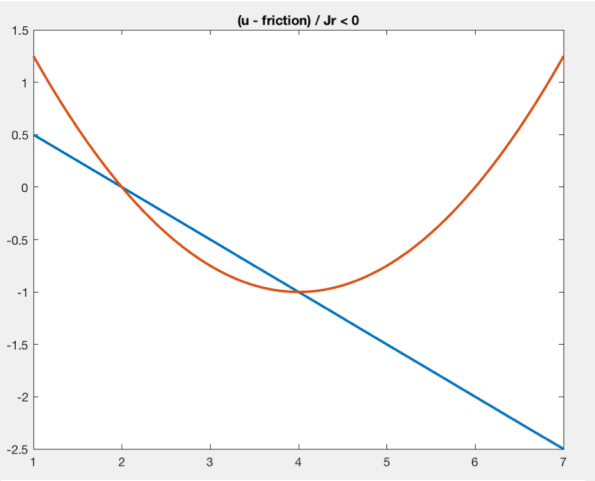
$$(\frac{(a2 * Jr * \Omega)}{c}) \leq d - pos)$$

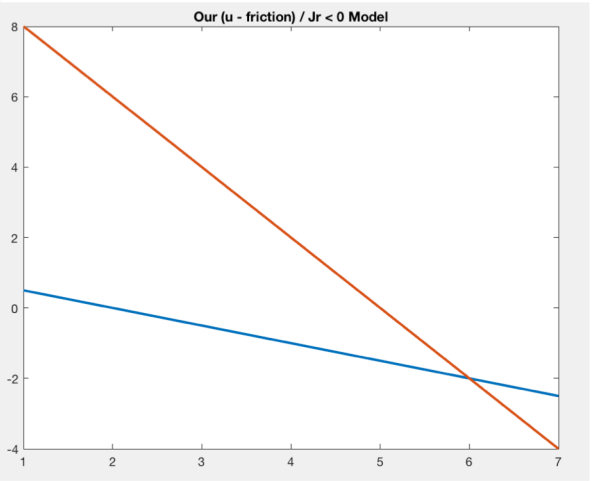
$$pos' = a1 * wob + a2 * \Omega$$

$$\frac{(a2 * (\Omega_0 + \frac{1}{Jr}(u - friction) * t) * Jr)}{c} \leq d - (pos_0 + a1 * wob * t + a2 * (\frac{1}{Jr}\frac{(u - friction) * t^2}{2} + \Omega_0 * t))$$

Weaker Bound (u - friction)/ Jr < 0







$$pos \leq pos_0 + a1 * wob * t + a2 * \Omega_0 * t$$

Conclusion

- We were able to successfully model a time-triggered smart dental drill
- We must always be ready for appropriate simplifications when modeling
- We hope with our model, dental drills can always be safely and effectively operated

Future Expansions

- Consider the drill rotary and the drill bit to be two separate parts with their own angular motion
- A more accurate friction model
- Consider not only the safety but the efficiency of the drill as well

Special Thanks

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- Irene Li
- Andrew Sogokon

Picture Credits

https://patents.google.com/patent/US3871097A/en

https://www.sic-invent.com/patients/sequence/

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