15-819M Data, Code, Decisions

Assignment 1 ($\sum 50$) due by Tue 9/29/2009

André Platzer

Carnegie Mellon University, Computer Science Department, Pittsburgh, PA aplatzer@cs.cmu.edu

Disclaimer: No solution will be accepted that comes without an **explanation**!

Exercise 1 Propositional Sequent Calculus (28p)

1. Which of the following formulas is true? Give a counterexample or a proof in sequent calculus.
   a) $(a \leftrightarrow b) \leftrightarrow (a \rightarrow \neg b) \wedge (\neg b \rightarrow a)$
   b) $(c \leftrightarrow d) \rightarrow ((a \leftrightarrow c) \leftrightarrow (a \leftrightarrow d))$
   c) $\neg a \rightarrow (a \rightarrow b)$
   d) $\Big(\big(((p \rightarrow q) \rightarrow (\neg r \rightarrow \neg s)) \rightarrow r\big) \rightarrow t\Big) \rightarrow \ \big((t \rightarrow p) \rightarrow (s \rightarrow p)\big)$
2. In at least two of the above cases, please send a KeY[1] proof file by email.
3. Give sequent proof rules for the following operators:
   a) XOR (exclusive-or)
   b) NAND (negated and)
   c) NOR (negated or)
   d) $A?B:C$ with the semantics

$$[\![A?B:C]\!]_I = \begin{cases} [\![B]\!]_I & \text{if } [\![A]\!]_I = \mathit{true} \\ [\![C]\!]_I & \text{if } [\![A]\!]_I = \mathit{false} \end{cases}$$

Exercise 2 Propositional Proofs (10p)

Is the following rule a replacement for the implication left rule?

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma, A, B \vdash \Delta}{\Gamma, A \rightarrow B \vdash \Delta}$$

What is the advantage of this rule? Is it a sound replacement? Is it a complete replacement for the implication left rule?

[1] http://www.key-project.org/download/releases/webstart/KeY.jnlp

Exercise 3 DPLL (12p)

1. Prove or disprove unsatisfiability of the conjunction of the following formulas with DPLL

$$
\begin{array}{l}
\neg a \vee \neg b \vee c \vee \neg d \\
\neg b \vee \neg c \\
b \vee \neg c \vee d \\
\neg a \vee c \\
\neg a \vee b \vee \neg d \\
a \vee d
\end{array}
$$

2. If the previous formulas are unsatisfiable, what is the simple-most formula to add or to remove to make the formulas satisfiable? If the previous formulas are satisfiable, what is the simple-most formula to add or to remove to make the formulas unsatisfiable? In each case, prove or disprove in DPLL.