



# Safe Reinforcement Learning via Formal Methods

**Nathan Fulton** and André Platzer

Carnegie Mellon University





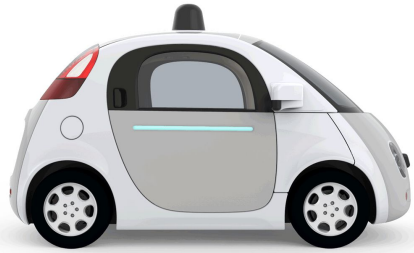
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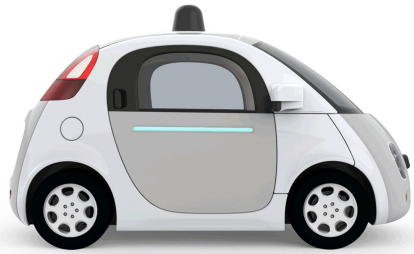


# Safety-Critical Systems



"How can we provide people with cyber-physical systems they can bet their lives on?" - Jeannette Wing

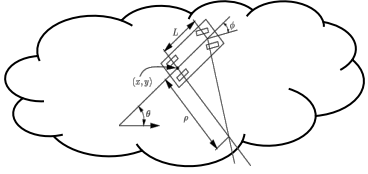
# Autonomous Safety-Critical Systems



How can we provide people with **autonomous** cyber-physical systems they can bet their lives on?

# Model-Based Verification

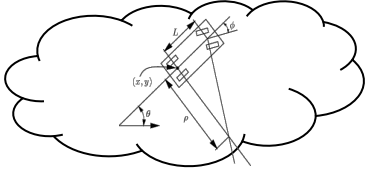
# Reinforcement Learning



$\varphi$

# Model-Based Verification

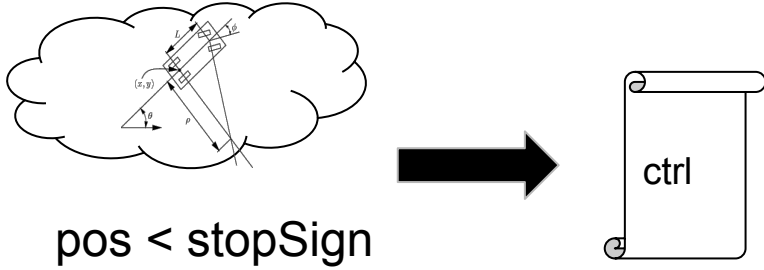
# Reinforcement Learning



pos < stopSign

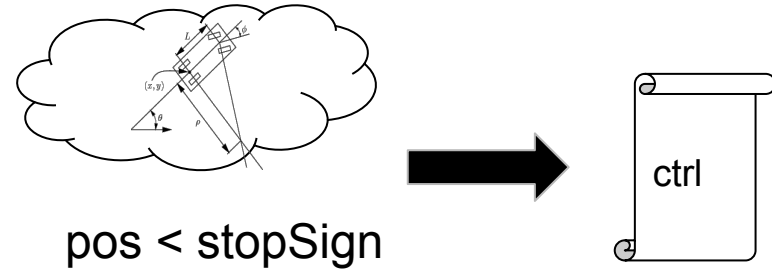
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**Approach:** prove that control software achieves a specification with respect to a model of the physical system.



# Model-Based Verification

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# Reinforcement Learning

## Benefits:

- Strong safety guarantees
- Automated analysis

# Model-Based Verification



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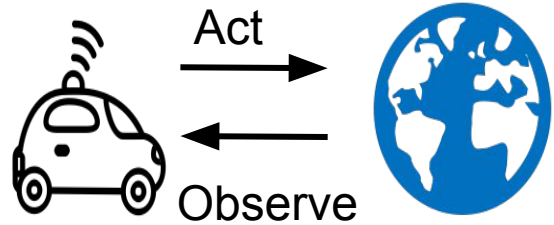
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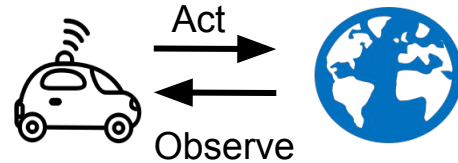
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# Reinforcement Learning



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- No need for complete model
- Optimal (effective) policies

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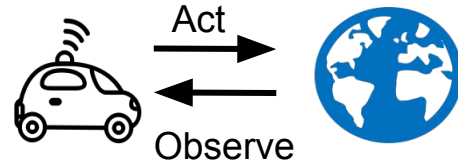
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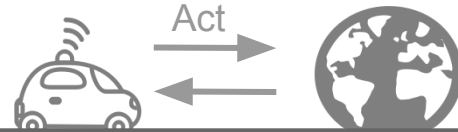
## Drawbacks:

- No strong safety guarantees
- Proofs are obtained and checked by hand
- Formal proofs = decades-long proof development

# Model-Based Verification



# Reinforcement Learning



**Goal: Provably correct reinforcement learning**

Benefits

- Strong safety guarantees
- Proofs are obtained and checked by hand

Drawbacks

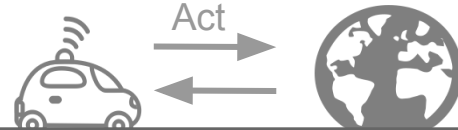
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# Model-Based Verification



# Reinforcement Learning



**Goal: Provably correct reinforcement learning**

- 1. Learn Safety**
- 2. Learn a Safe Policy**
- 3. Justify claims of safety**

Benefit

- Safety
- Assurance

Drawback

- Control policies are typically non-deterministic: answers “what is safe”, not “what is useful”
- Assumes accurate model

- No strong safety guarantees
- Proofs are obtained and checked by hand
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Model  
s

# Model-Based Verification

Accurate, analyzable models often exist!

```
{  
  {?safeAccel; accel U brake U ?safeTurn; turn};  
  {pos' = vel, vel' = acc}  
}*
```

# Model-Based Verification

**Accurate**, analyzable models often exist!

```
{  
  {?safeAccel; accel U brake U ?safeTurn; turn};  
  {pos' = vel, vel' = acc}  
}*  
      Continuous motion  
      discrete control
```

The diagram illustrates a hybrid system model. The top line is a discrete control logic: `{?safeAccel; accel U brake U ?safeTurn; turn};`. The middle line is a continuous motion equation: `{pos' = vel, vel' = acc}`. A horizontal bracket underlines the entire model, with an arrow pointing to the text "discrete control". A second horizontal bracket underlines only the continuous motion equation, with the text "Continuous motion" below it.

# Model-Based Verification

**Accurate**, analyzable models often exist!

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{?safeAccel; accel U brake U ?safeTurn; turn};

{pos' = vel, vel' = acc}

}\*

Continuous motion

discrete, ***non-deterministic***  
control

# Model-Based Verification

**Accurate, analyzable** models often exist!

```
init → [{  
    { ?safeAccel; accel  U brake  U ?safeTurn; turn};  
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}*] pos < stopSign
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formal verification gives strong safety guarantees

```
init → [{  
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=

- **Computer-checked proofs of safety specification.**

# Model-Based Verification

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- Computer-checked proofs of safety specification
- Formal proofs mapping model to runtime monitors



# Model-Based Verification Isn't Enough

**Perfect**, analyzable models don't exist!

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How to implement?

{

{ ?safeAccel; accel U brake U ?safeTurn; turn};

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Only accurate sometimes

# Model-Based Verification Isn't Enough

**Perfect**, analyzable models don't exist!

How to implement?

{

{ ?safeAccel; accel  brake  ?safeTurn; turn};

{dx'=w\*y, dy'=-w\*x, ...}

}\*

Only accurate sometimes

# Our Contribution

**Justified Speculative Control** is an approach toward provably safe reinforcement learning that:

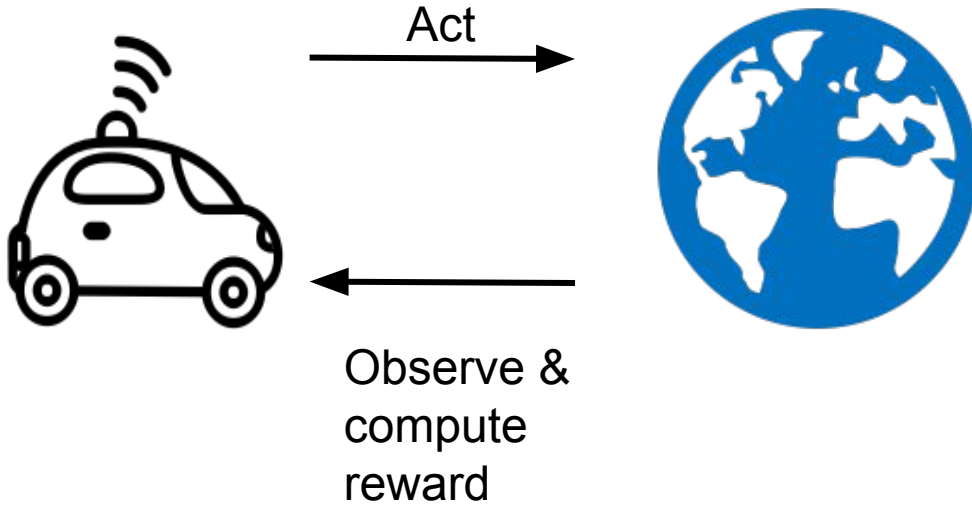
1. learns to resolve non-determinism without sacrificing formal safety results

# Our Contribution

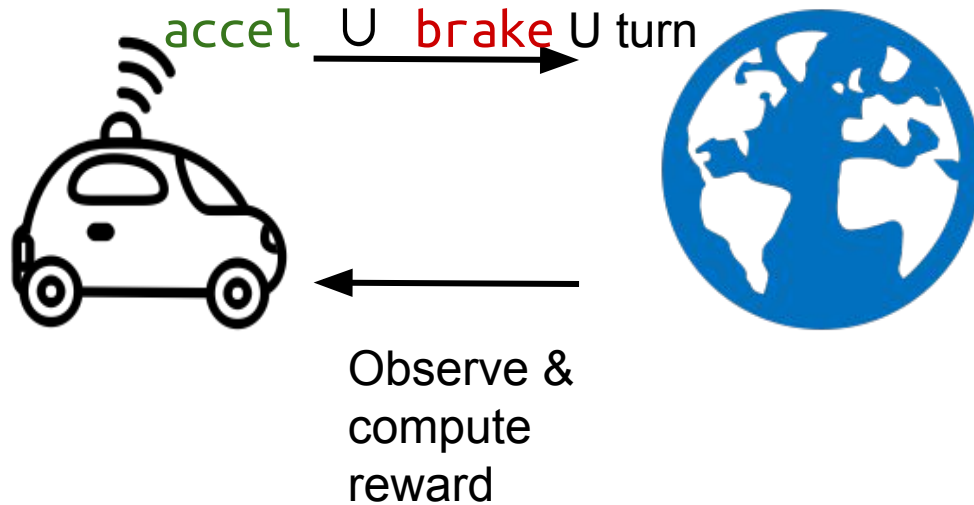
**Justified Speculative Control** is an approach toward provably safe reinforcement learning that:

1. learns to resolve non-determinism without sacrificing formal safety results
2. allows and directs speculation whenever model mismatches occur

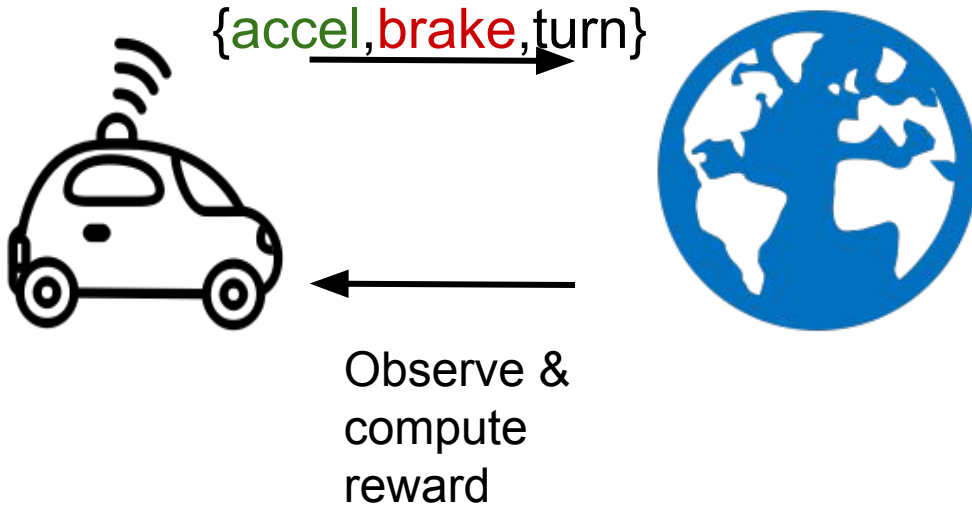
# Learning to Resolve Non-determinism



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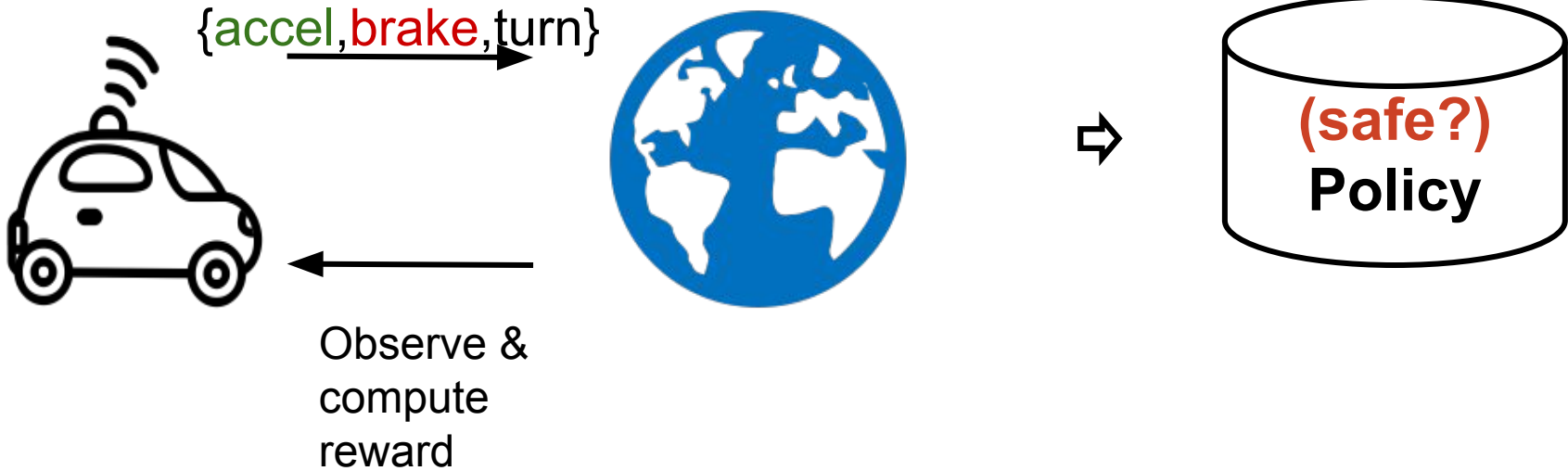




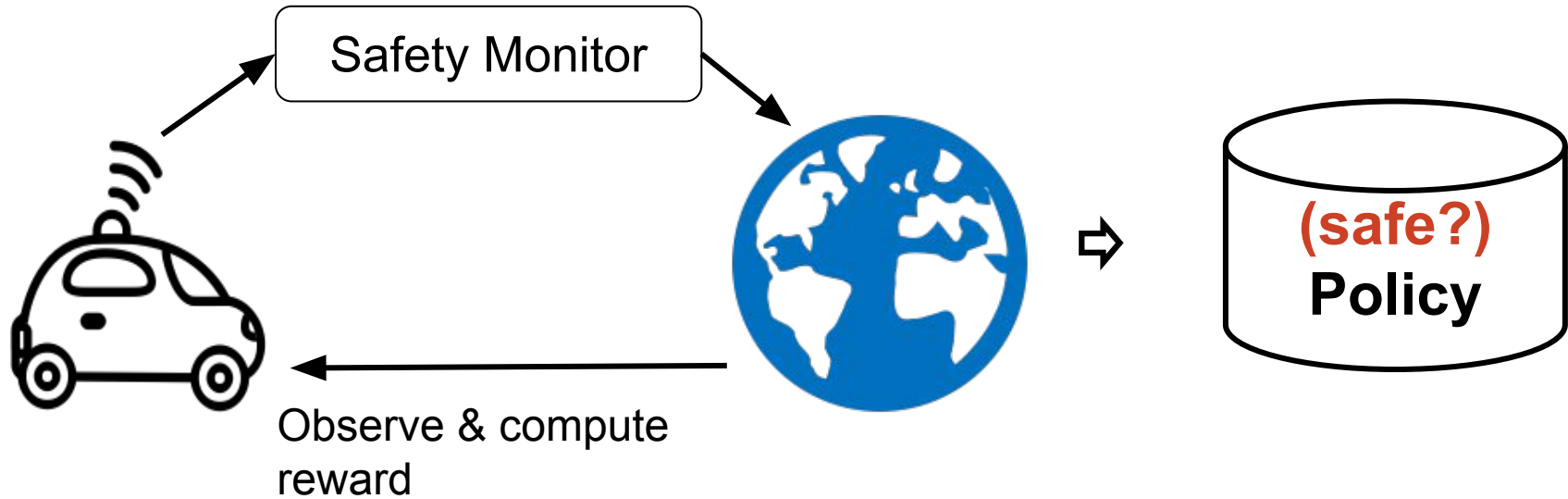
# Learning to Resolve Non-determinism



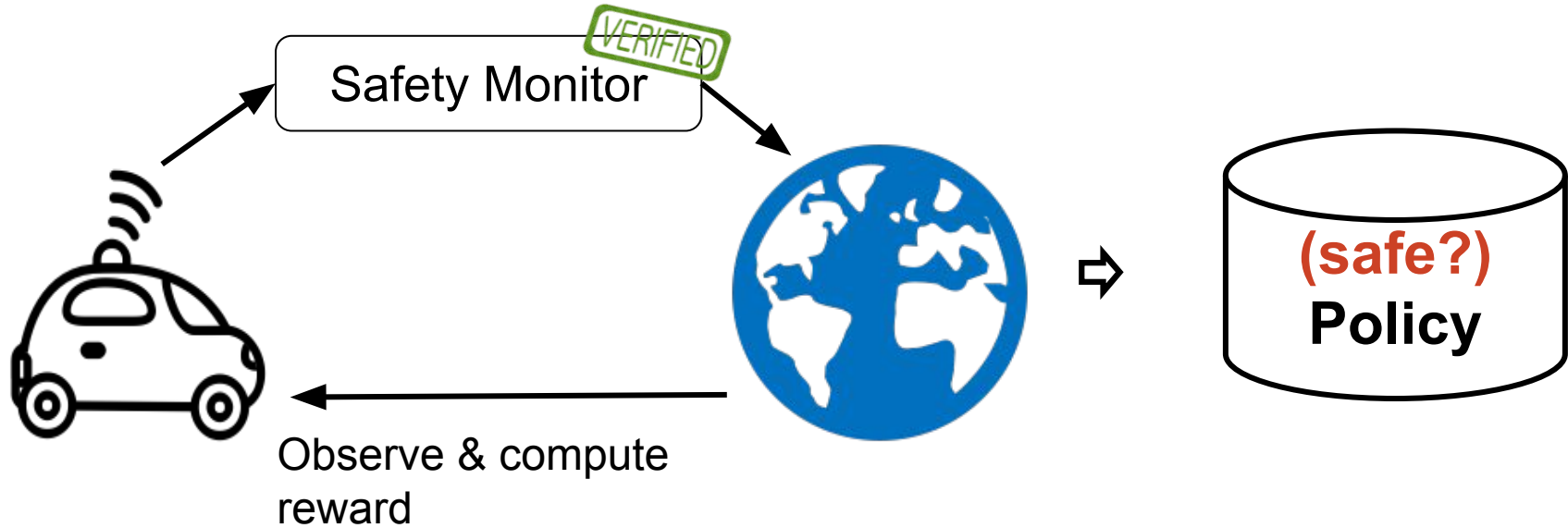
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# Learning to **Safely** Resolve Non-determinism

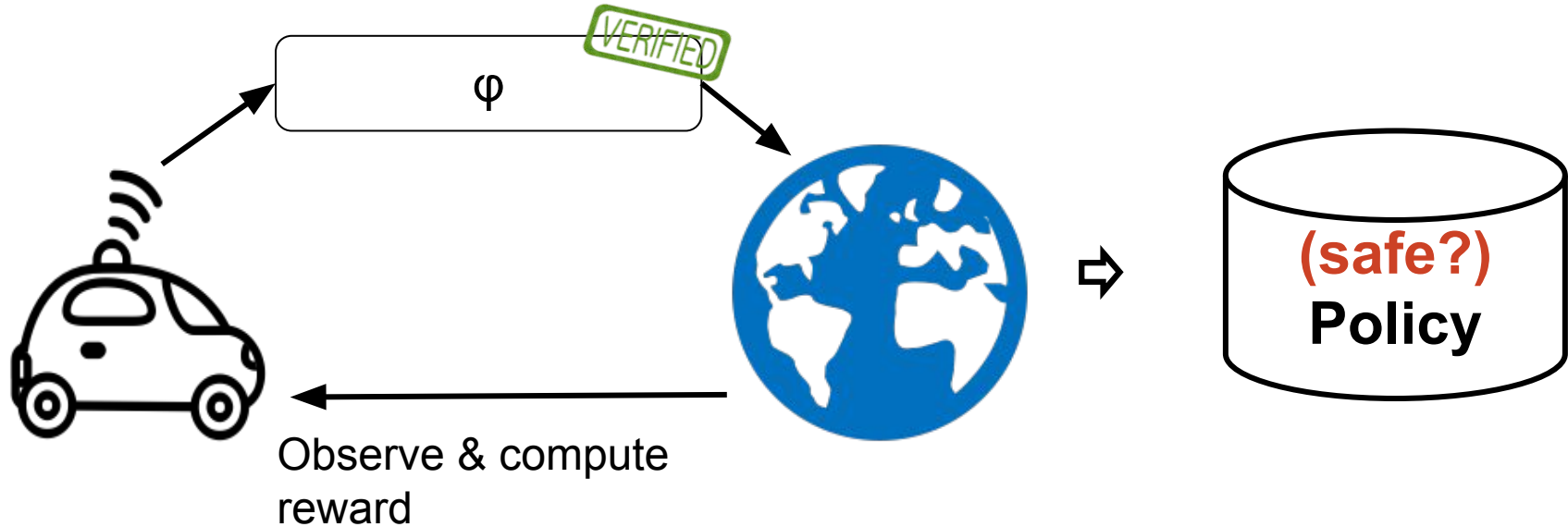


# Learning to **Safely** Resolve Non-determinism



**VERIFIED**  $\neq$  "Trust Me"

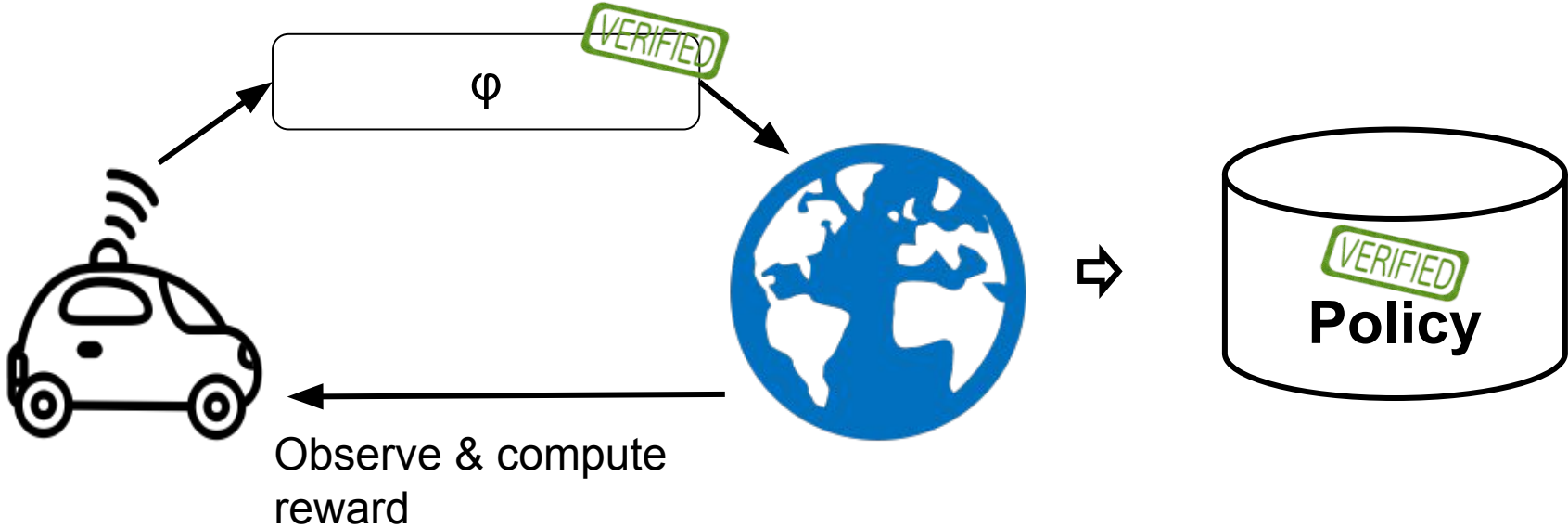
# Learning to **Safely** Resolve Non-determinism



Use a theorem prover to prove:

$$(\text{init} \rightarrow [\{\{\text{accel} \cup \text{brake}\}; \text{ODEs}\}^*](\text{safe})) \leftrightarrow \varphi$$

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# Learning to **Safely** Resolve Non-determinism



**Main Theorem:** If the ODEs are accurate, then our formal proofs transfer from the non-deterministic model to the learned **(deterministic) policy**

Use a theorem prover to prove:

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# Learning to **Safely** Resolve Non-determinism



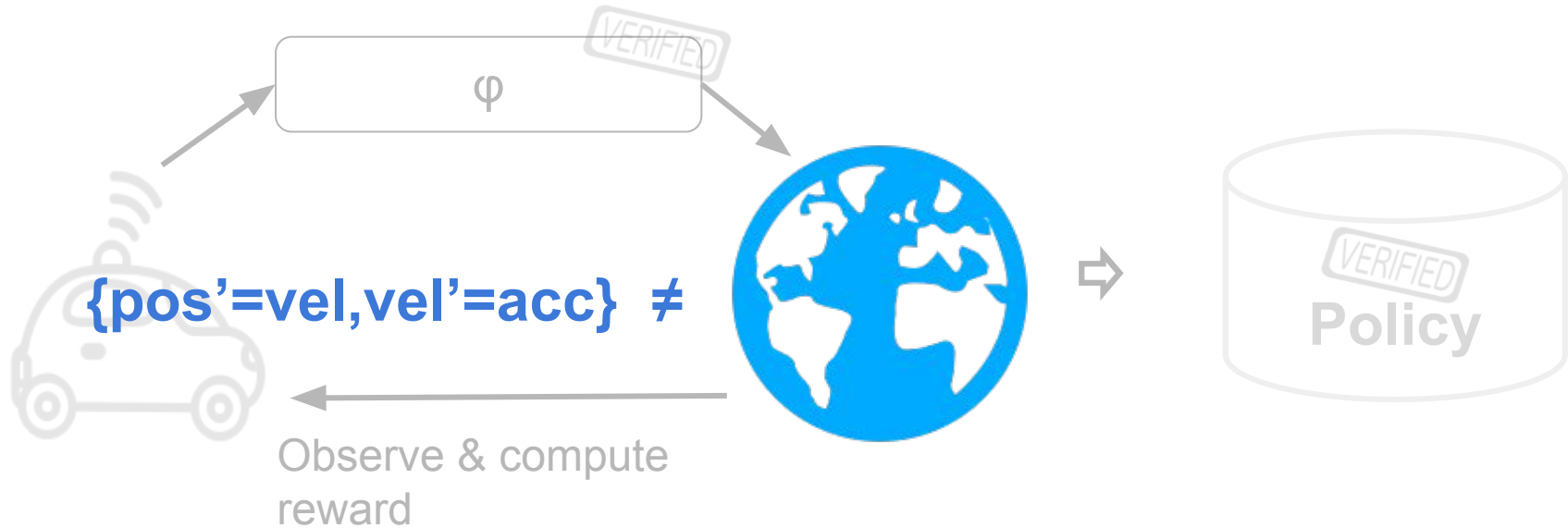
**Main Theorem:** If the ODEs are accurate, then our formal proofs transfer from the non-deterministic model to the learned **(deterministic) policy** via the model monitor.

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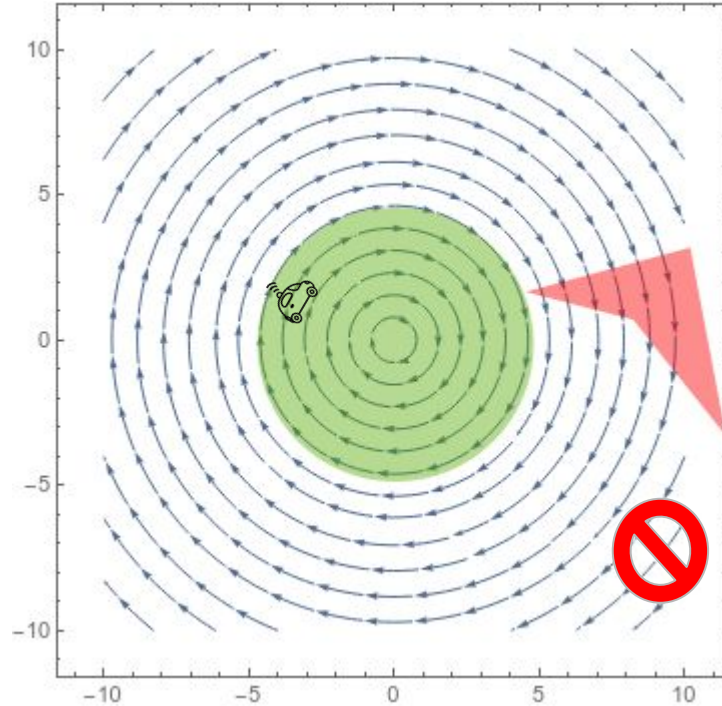
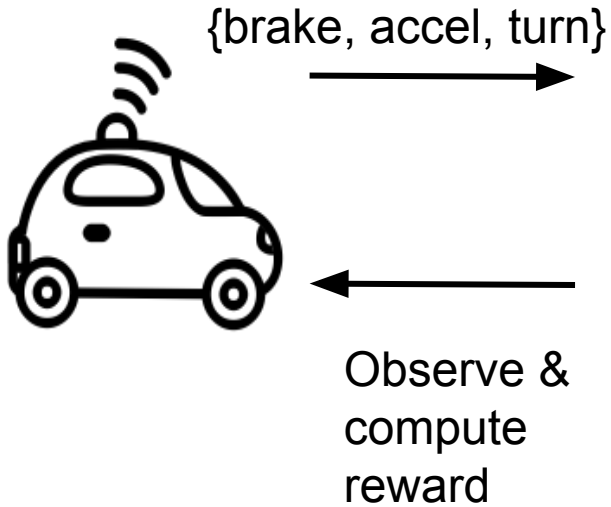


# What about the physical model?



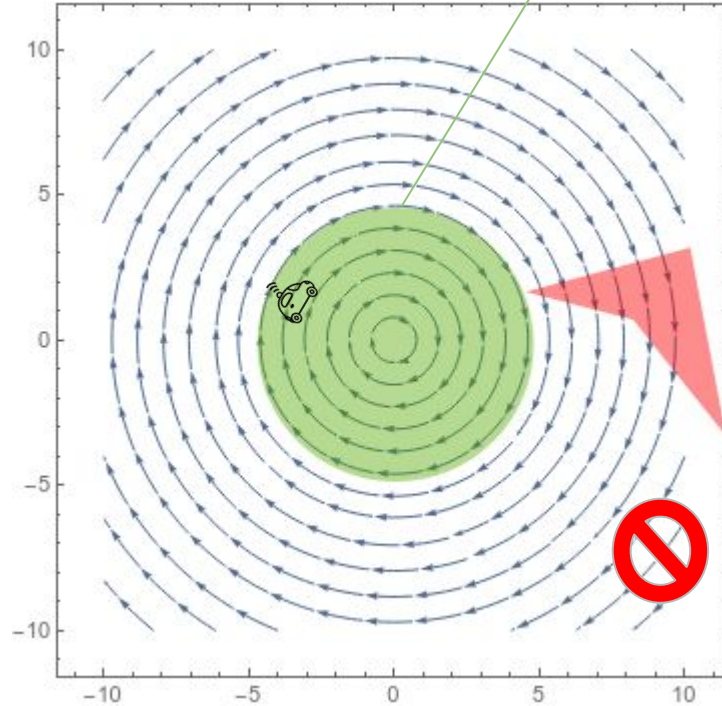
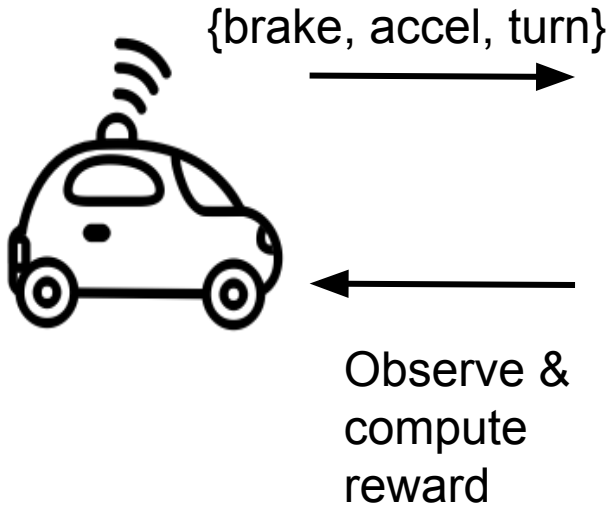
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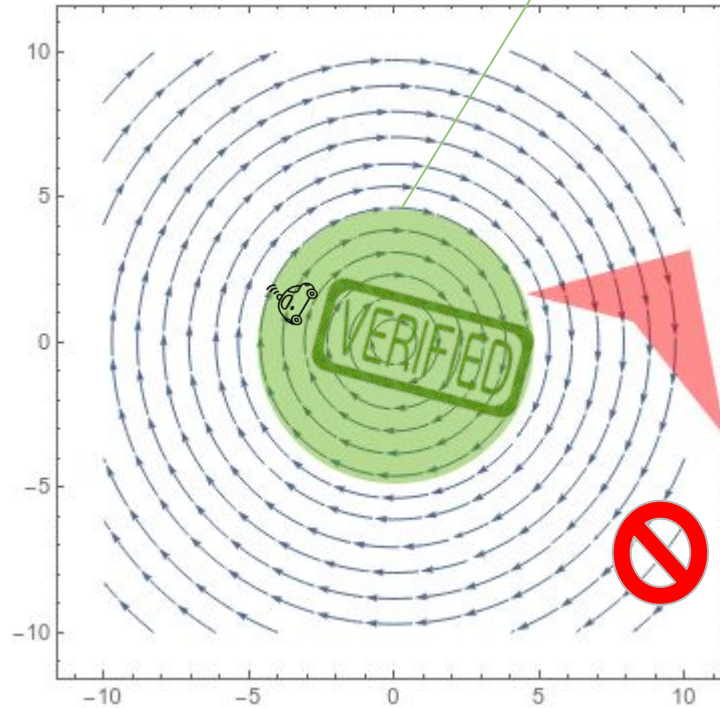
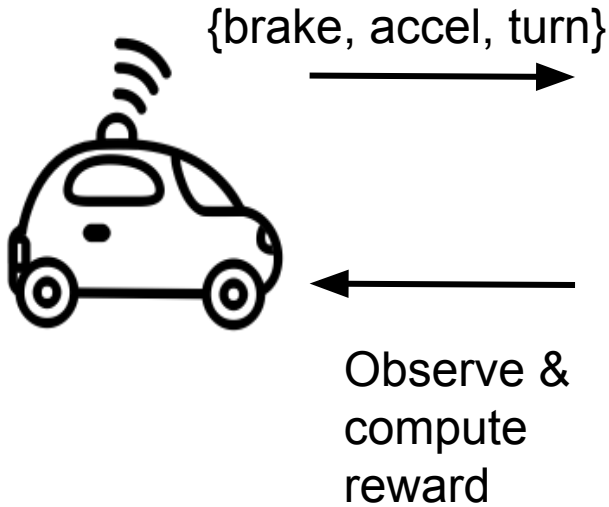
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Model is accurate.

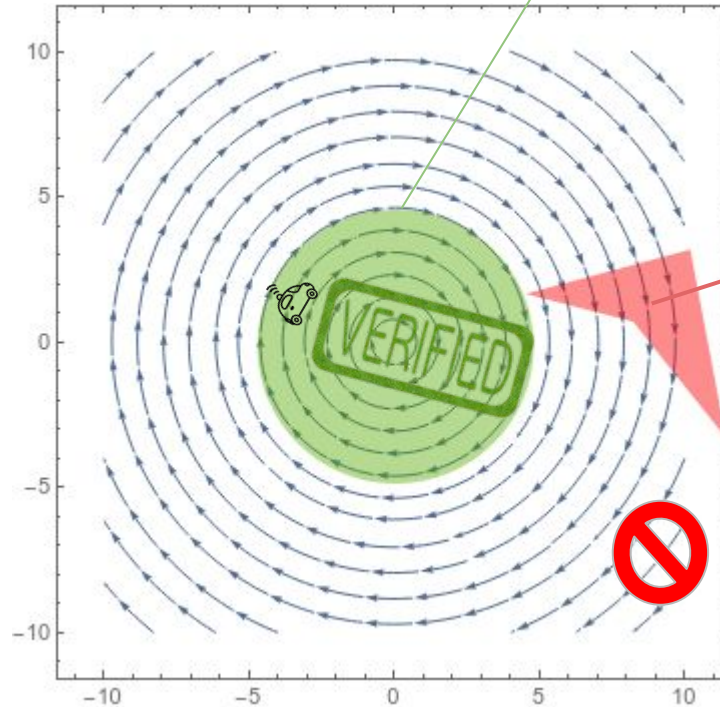
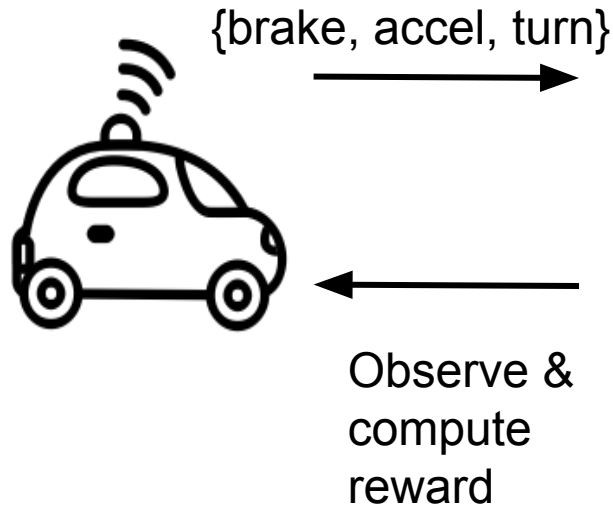


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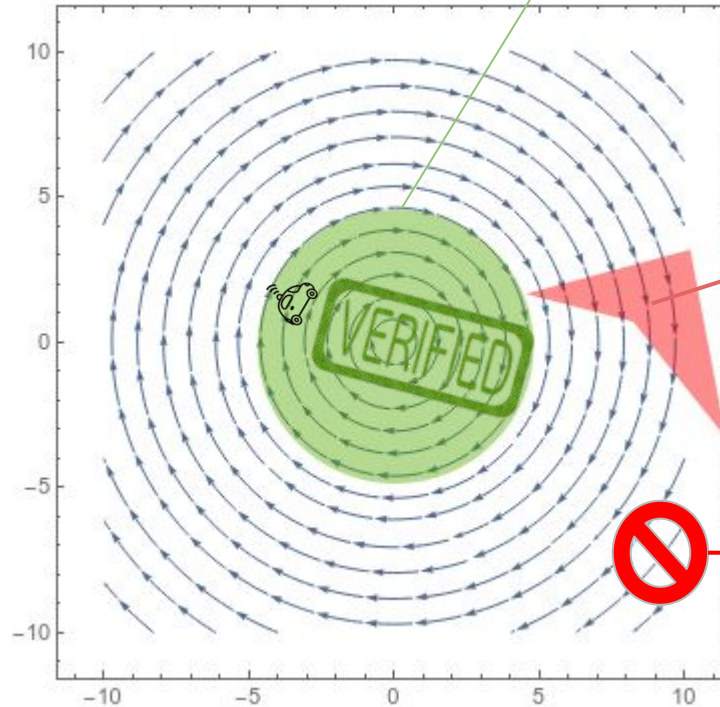
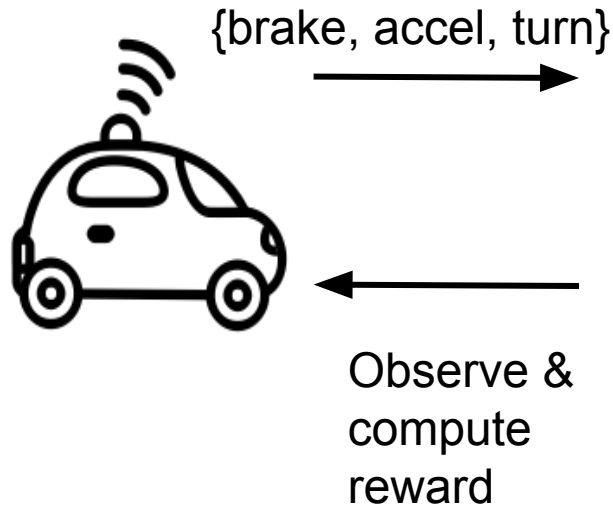
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Model is accurate.

Model is inaccurate

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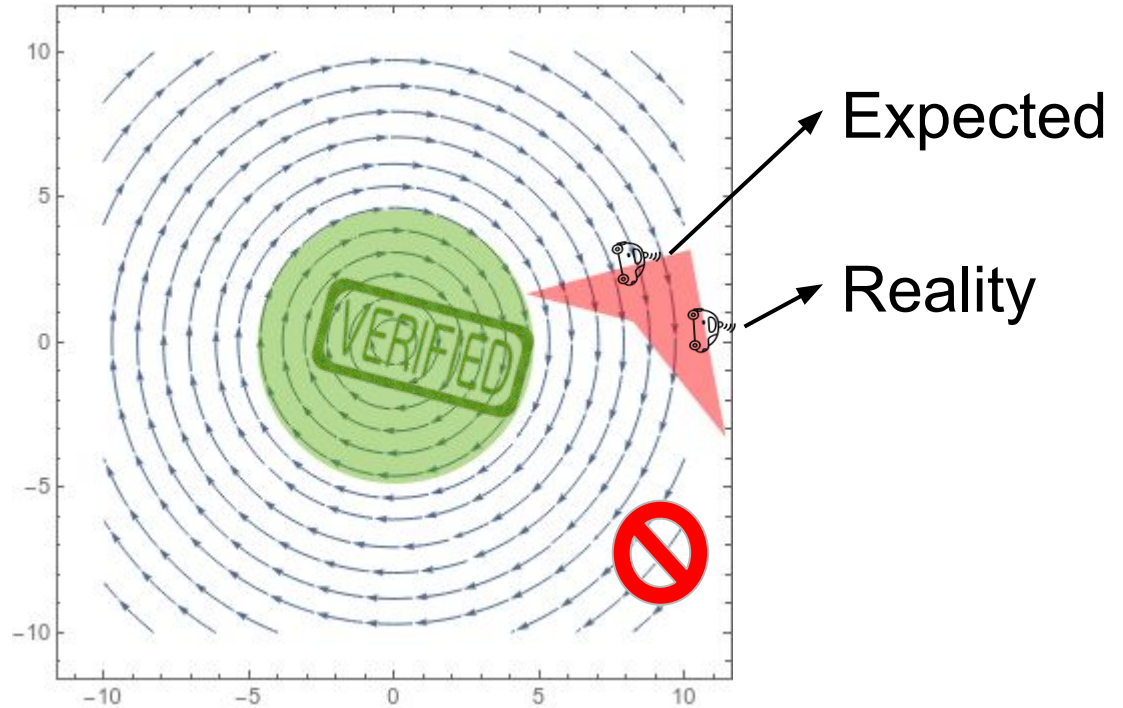
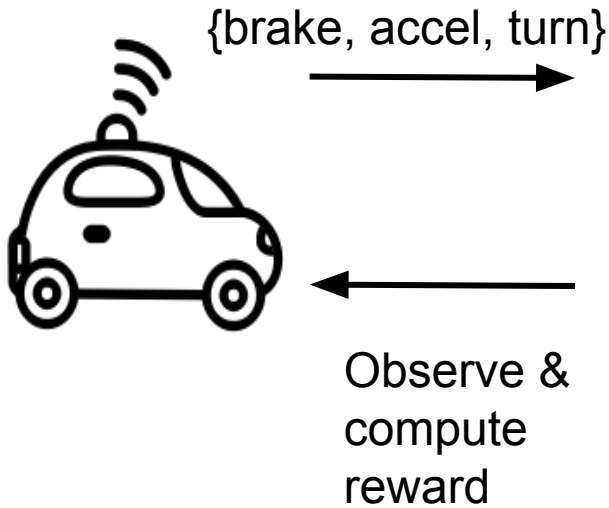


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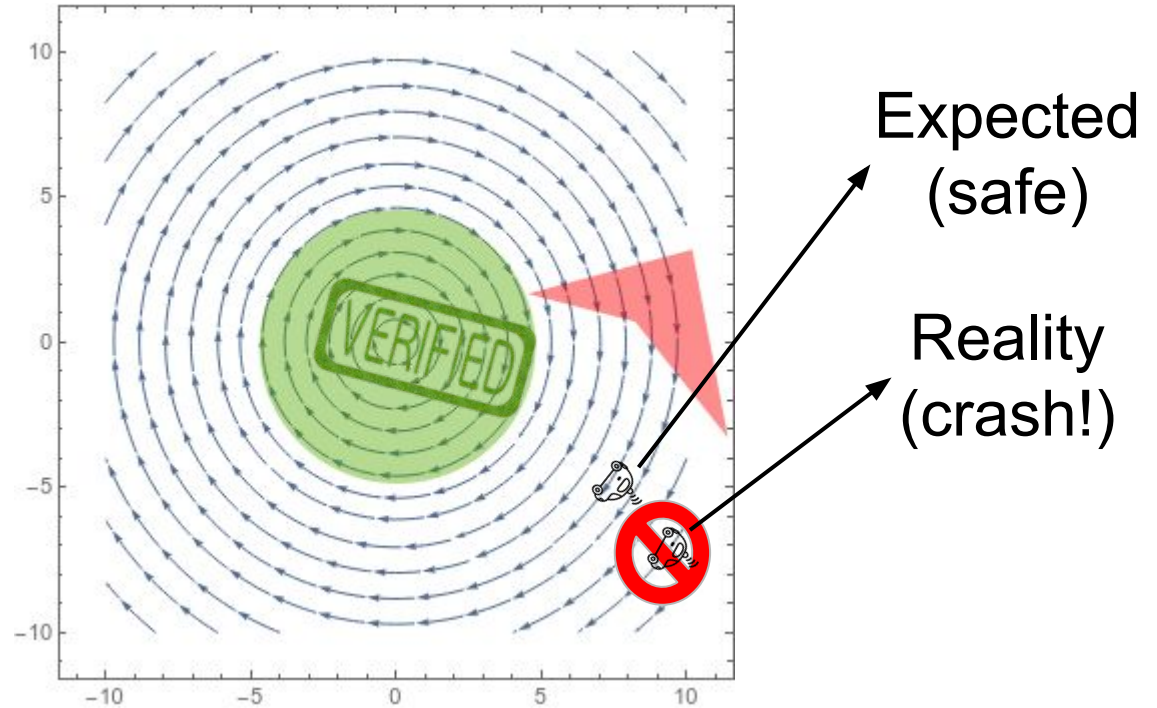
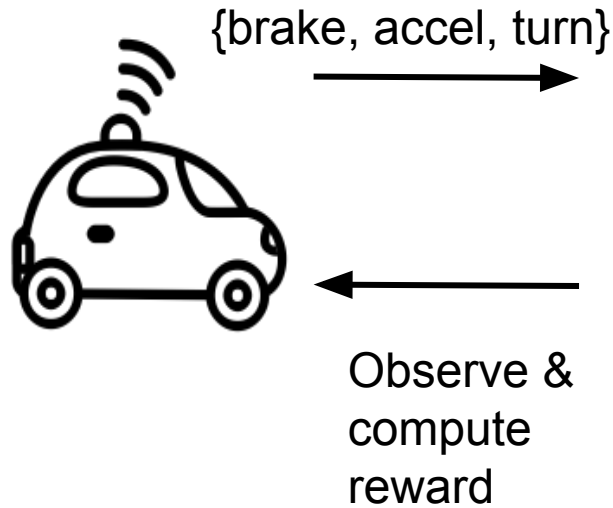
Model is inaccurate

Obstacle!

# What About the Physical Model?

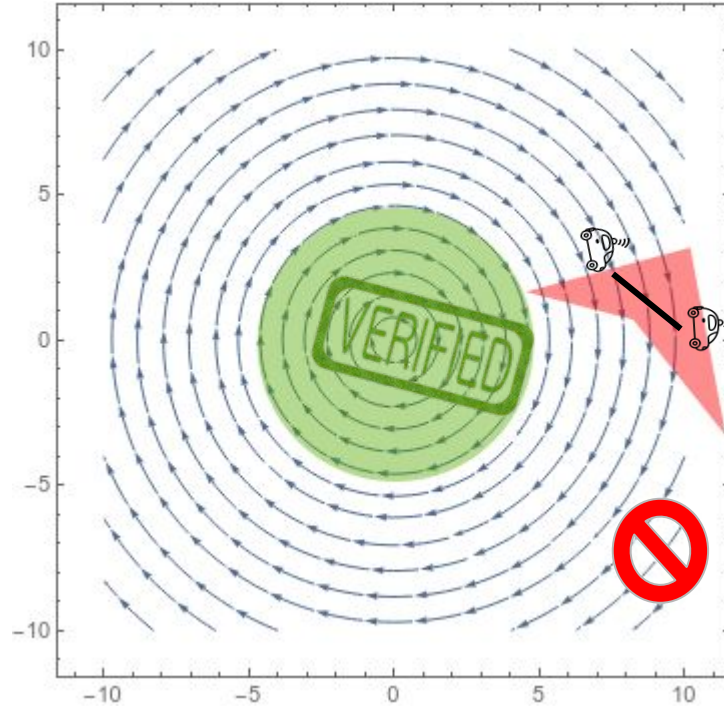
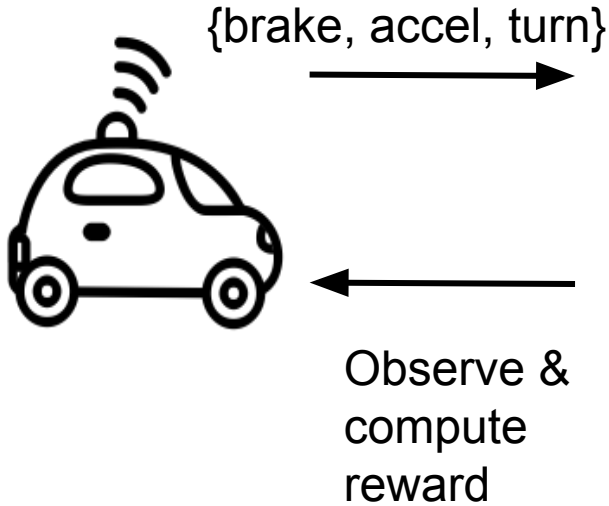


# Speculation is Justified





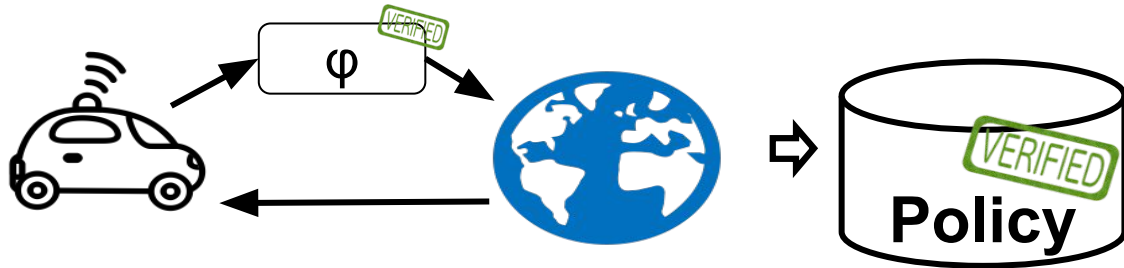
# Leveraging Verification Results to Learn Better



Use a real-valued version of the model monitor as a reward signal

# Conclusion

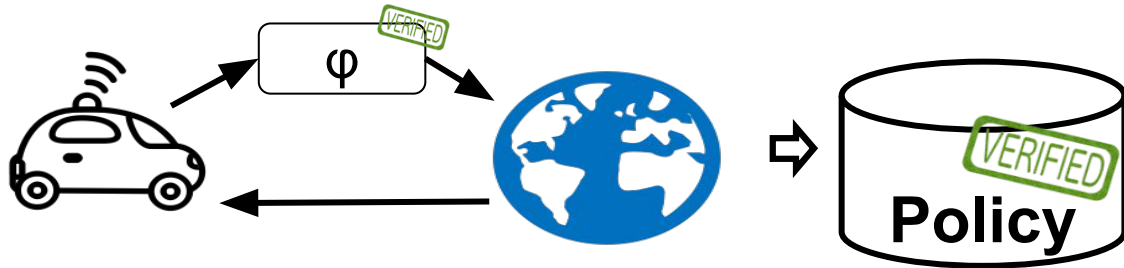
**Justified Speculative Control** provides the best of logic and learning:



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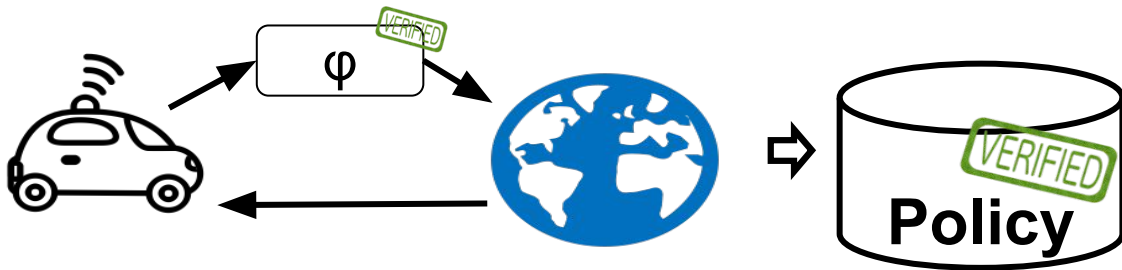
- Formally model the control system (**control + physics**)



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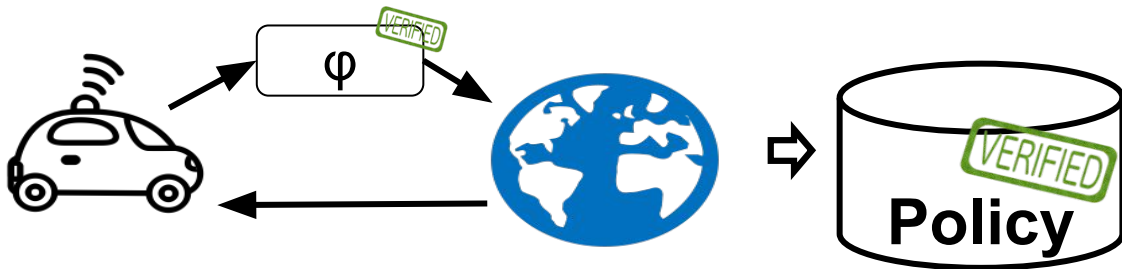
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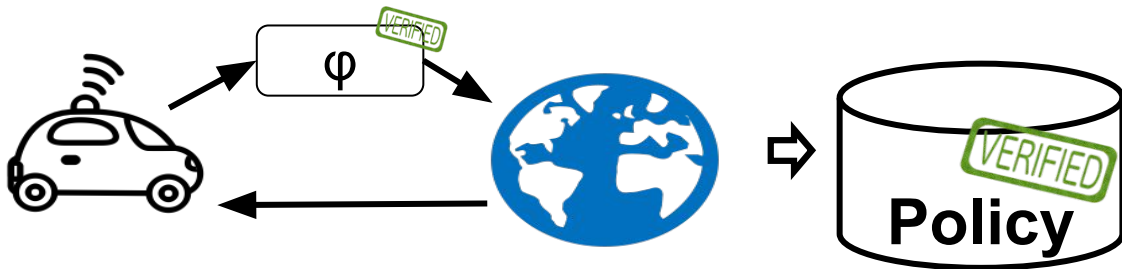
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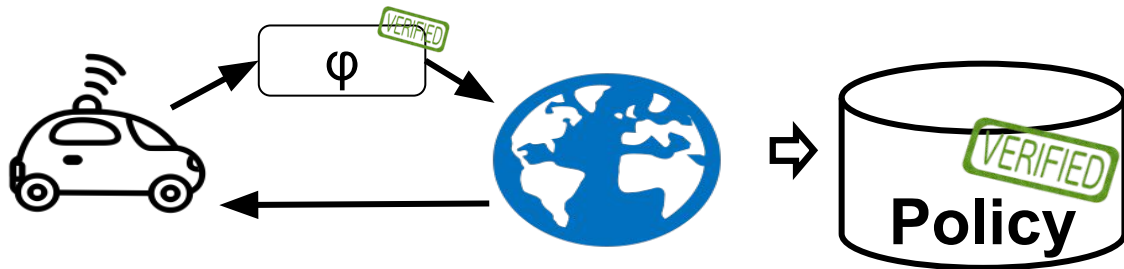
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- Unsafe **speculation is justified** when model deviates from reality, but **verification results can still be helpful!**

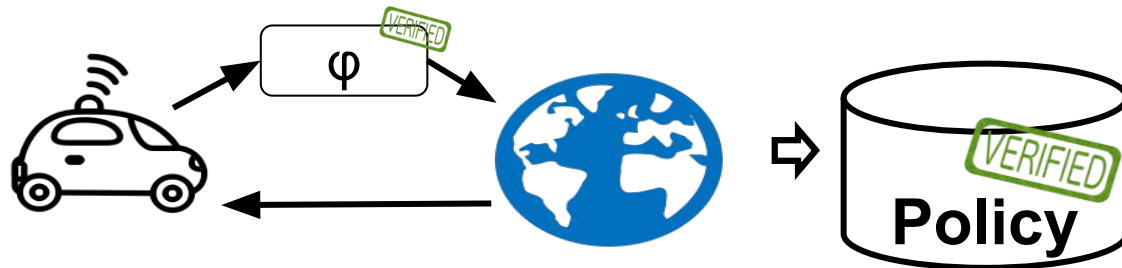


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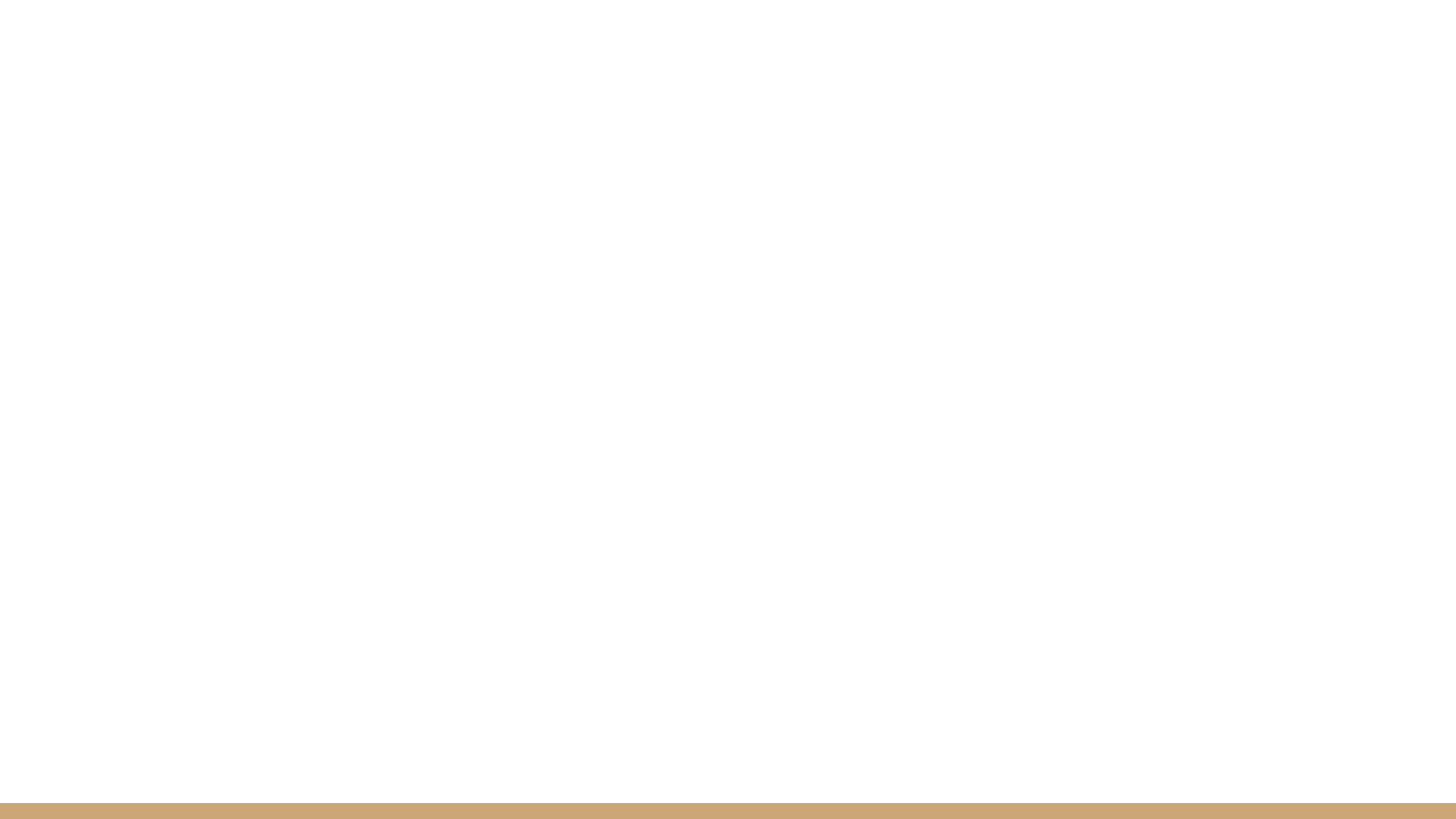


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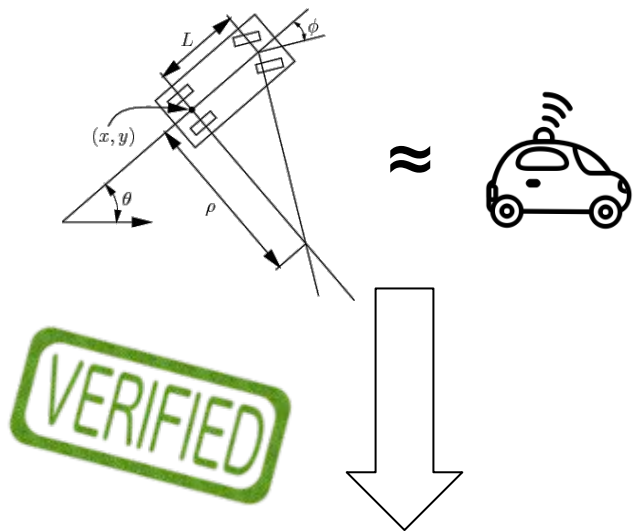




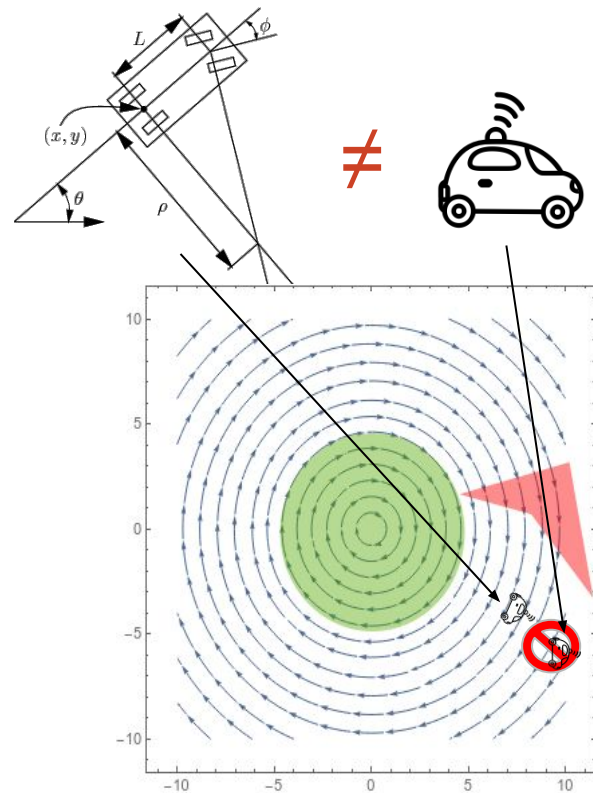




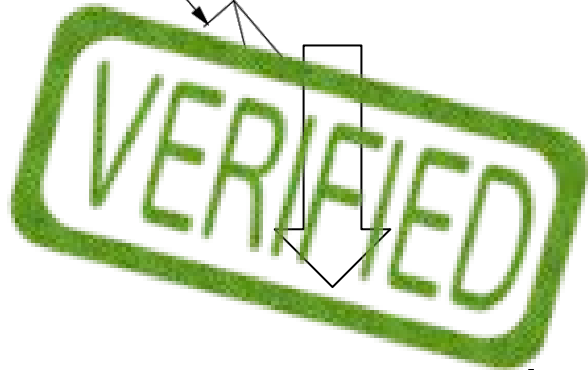
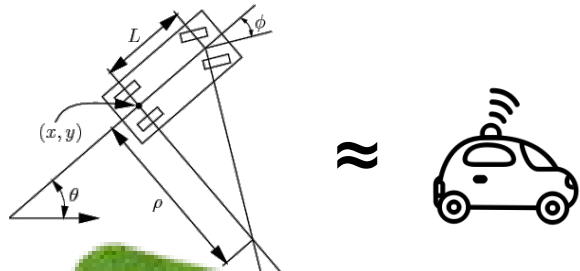
# Justified Speculative Control



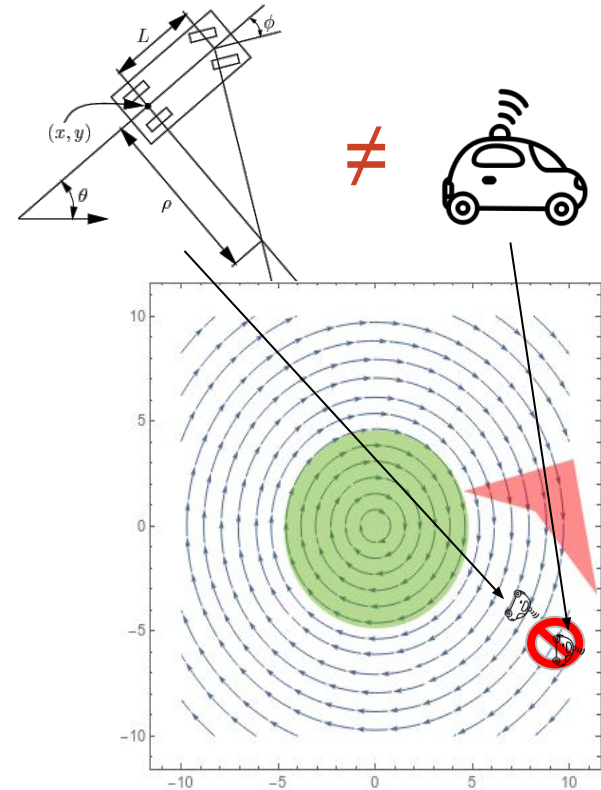
Learn over a constrained  
action space



# Justified Speculative Control



Learn over a constrained  
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# Safe Reinforcement Learning?



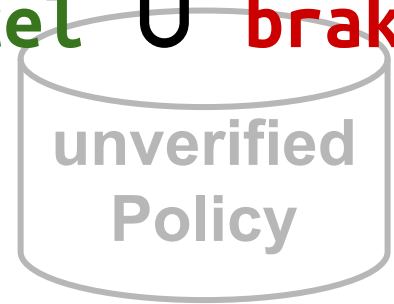
Policy deviates from model:

1. Policy is deterministic, verification result is set-valued.

Some Actions Aren't Always Safe  
 $\{\text{accel}, \text{brake}, \text{turn}\} \neq ?\text{safeAccel}; \text{accel} \cup \text{brake}$



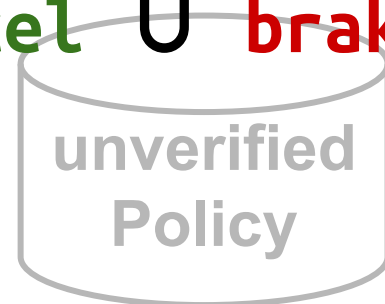
Observe &  
compute reward



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# Safe Reinforcement Learning?



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# Physical Models are Approximations

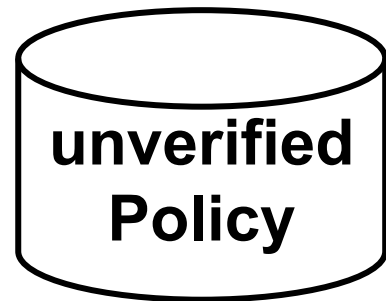


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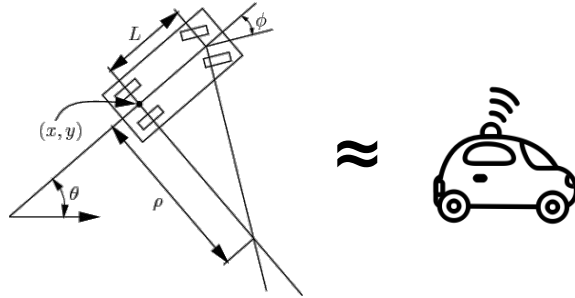
1. Policy is deterministic, verification result is set-valued.
2. Environment may not be accurately modeled.

# Safety resolving non-determinism

?safeAccel; accel U brake  $\neq$



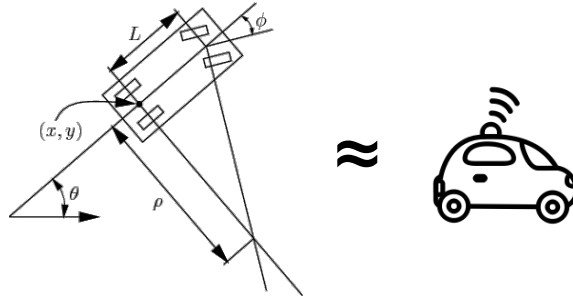
# Sandboxing Reinforcement Learning



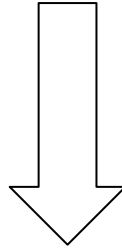
“Accurate modulo determinism”

init  $\rightarrow$  [ $\boxed{\{\text{accel} \cup \text{brake}\}}$ ; t:=0; continuousMotion ]\*(safe)

# Sandboxing Reinforcement Learning

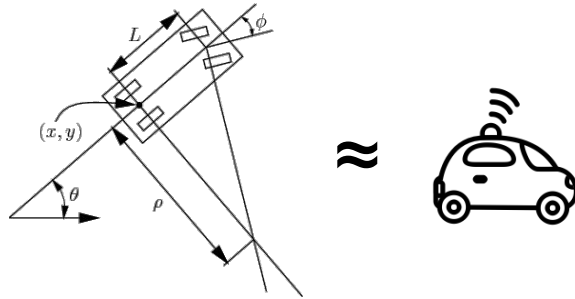


“Accurate modulo determinism”



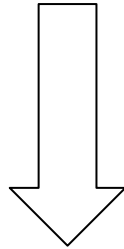
Learn over a constrained  
action space

# Sandboxing Reinforcement Learning



“Accurate modulo determinism”

VERIFIED



Learn over a constrained  
action space

# Sandboxing Reinforcement Learning



Theorem: **If the physical model is accurate** then verification results are preserved during learning and by learned policies.

# Sandboxing Reinforcement Learning



init  $\rightarrow$  [ $\{ \text{accel} \cup \text{brake} \}; t:=0; \boxed{\text{continuousMotion}}^* \text{](safe)}$

**Theorem: If the physical model is accurate** then verification results are preserved during learning and by learned policies.



# Sandboxing Reinforcement Learning



```
init → [{ {accel U brake}; t:=0; continuousMotion }*](safe)
```

Theorem: If the physical model is accurate then **verification results are preserved during learning and by learned policies.**

# ~~Sandboxing~~ Safe Reinforcement Learning

**Theorem 1** (JSCGeneric Explores Safely in Modeled Environments). Assume a valid safety specification

$$\models \text{init} \rightarrow [\{\text{ctrl}; \text{plant}\}^*] \text{safe} \quad (3)$$

i.e., any repetition of  $\{\text{ctrl}; \text{plant}\}$  starting from a state in  $\text{init}$  will end in a state described by  $\text{safe}$ . Then  $u_i(s_i) \models \text{safe}$  for all  $u_i, s_i$  satisfying the learning process for

$$(\text{init}, (S, A, R, E), \text{choose}, \text{update}, \text{done}, \text{CM}, \text{MM})$$

$\text{init} \rightarrow$

where  $\text{CM}$  and  $\text{MM}$  are the controller and model monitor for

$$\text{init} \rightarrow [\{\text{ctrl}; \text{plant}\}^*] \text{safe}.$$

Theorem

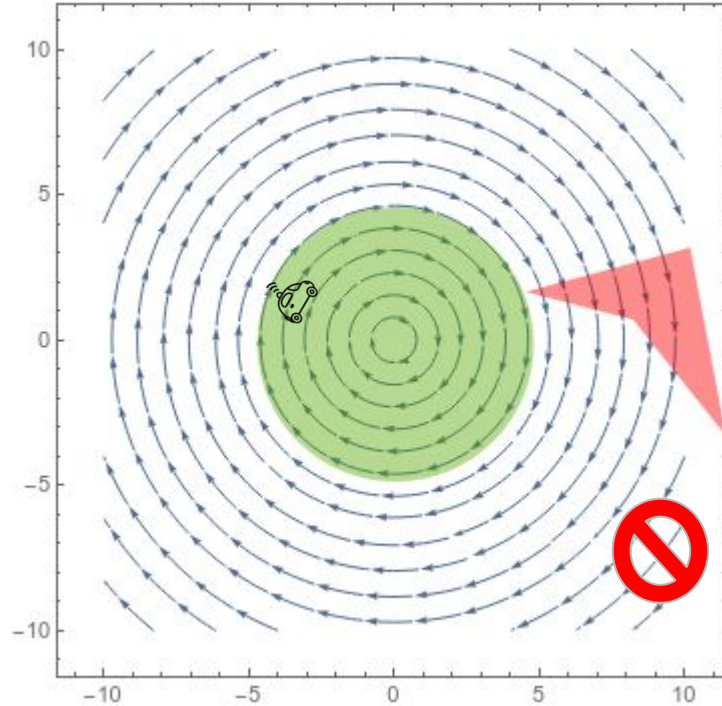
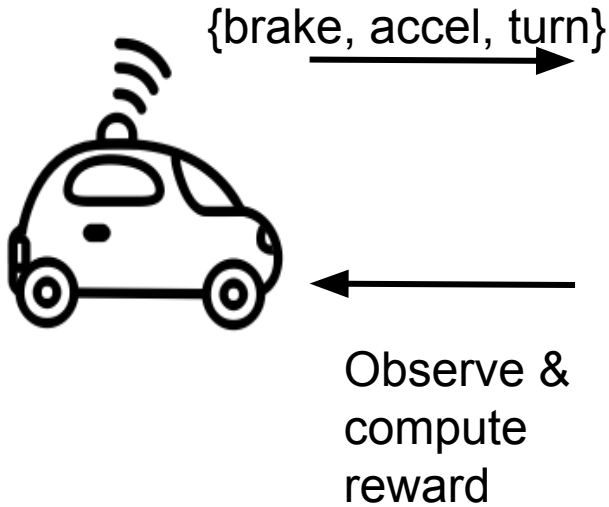
results are preserved by learned policies.



(safe)

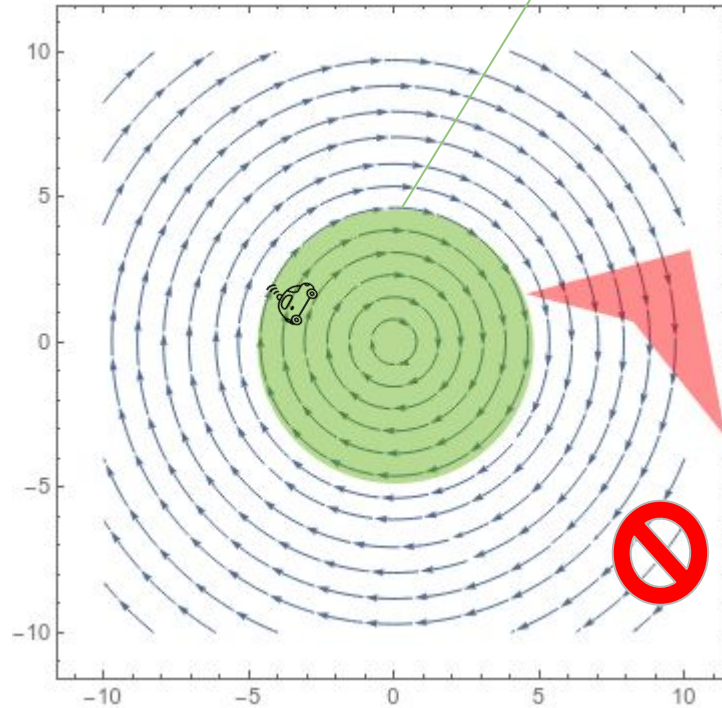
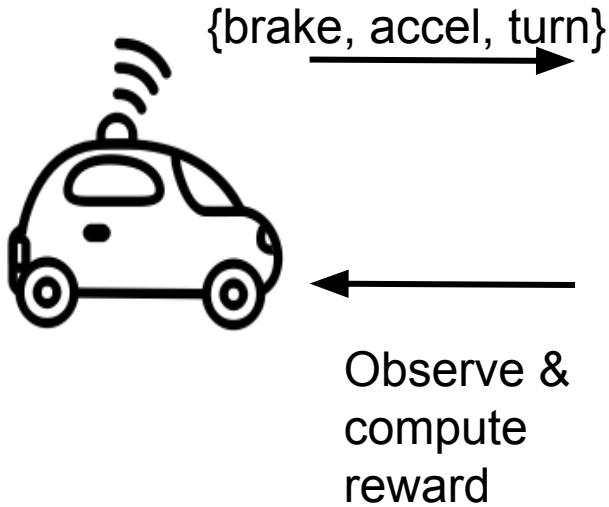
ation

# What About the Physical Model?



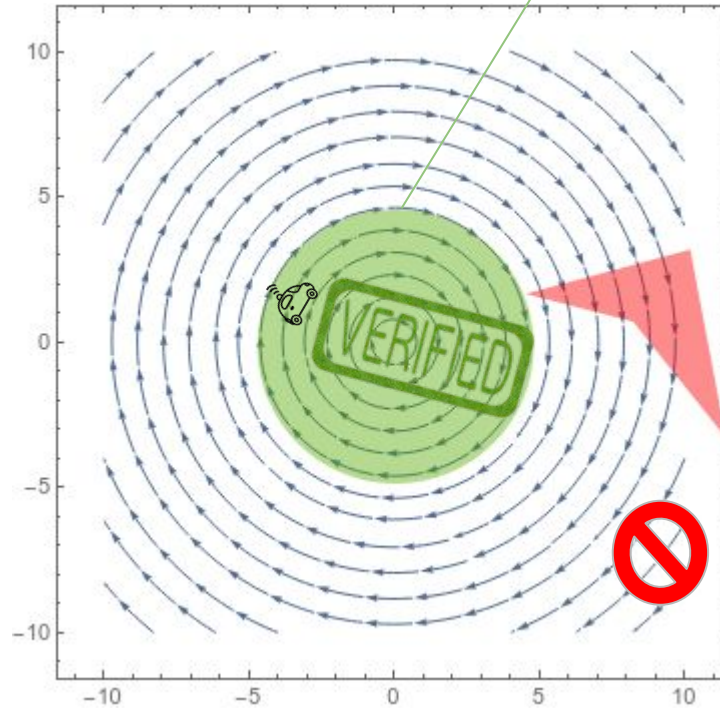
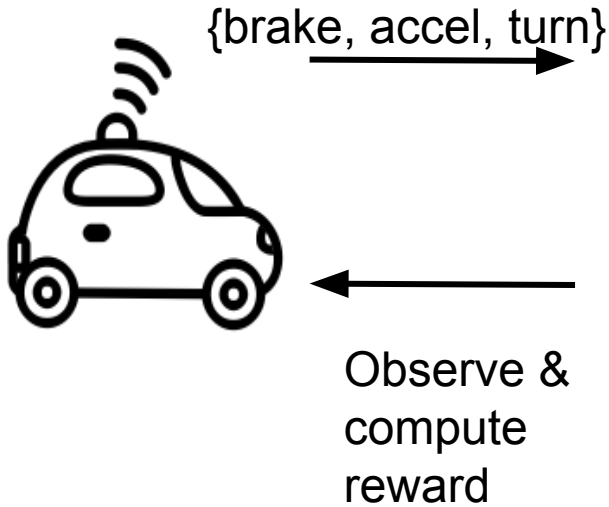
# What About the Physical Model?

Model is accurate.

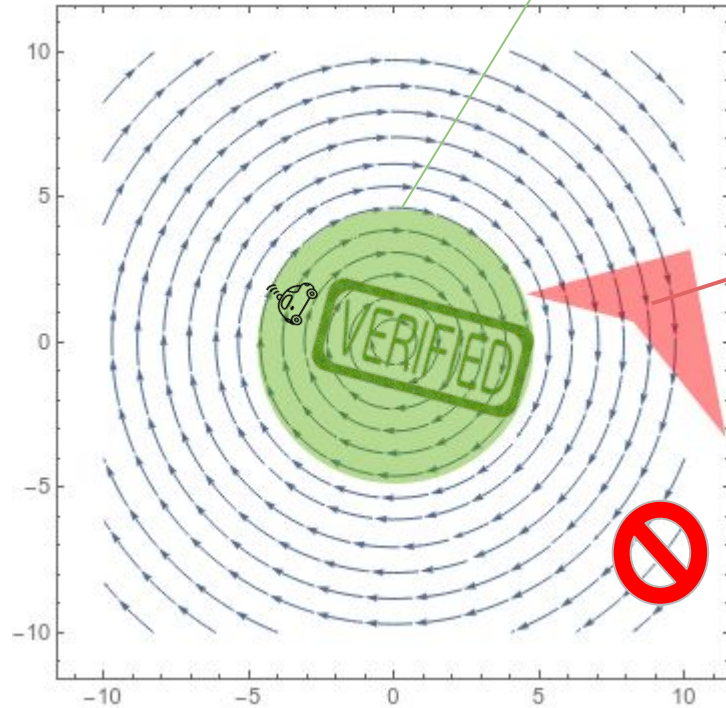
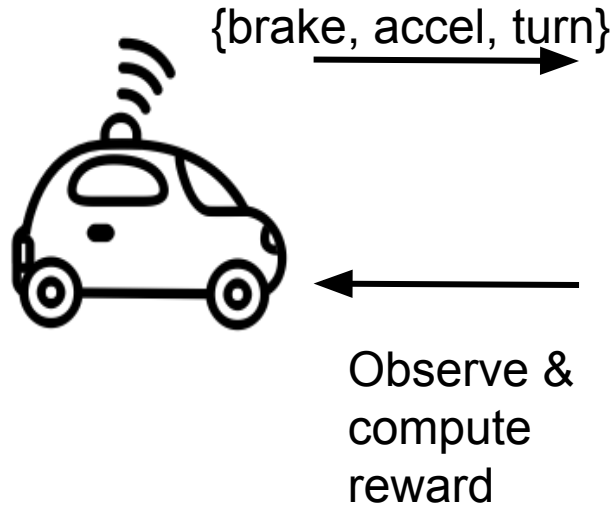


# What About the Physical Model?

Model is accurate.



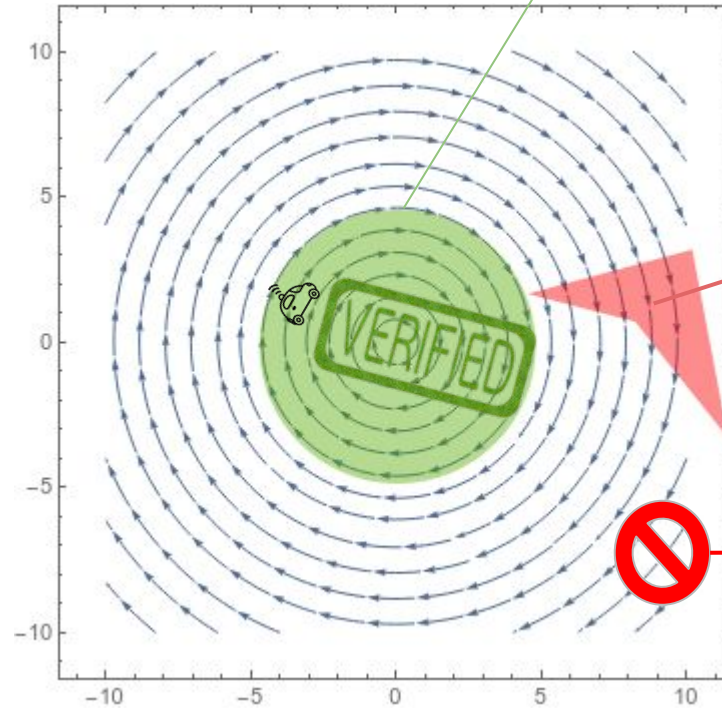
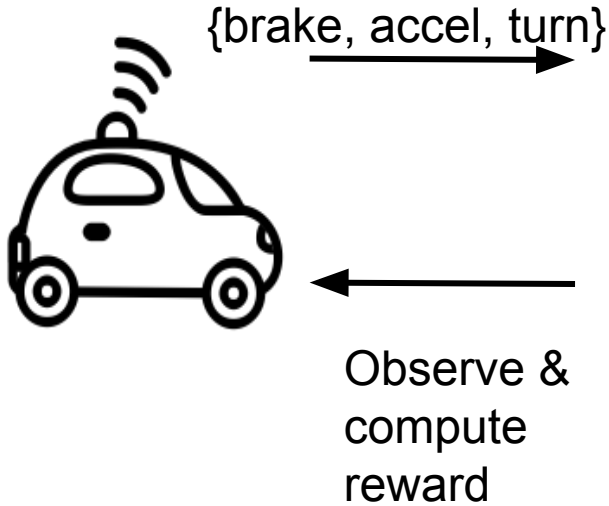
# What About the Physical Model?



Model is correct.

Model is inaccurate

# What About the Physical Model?

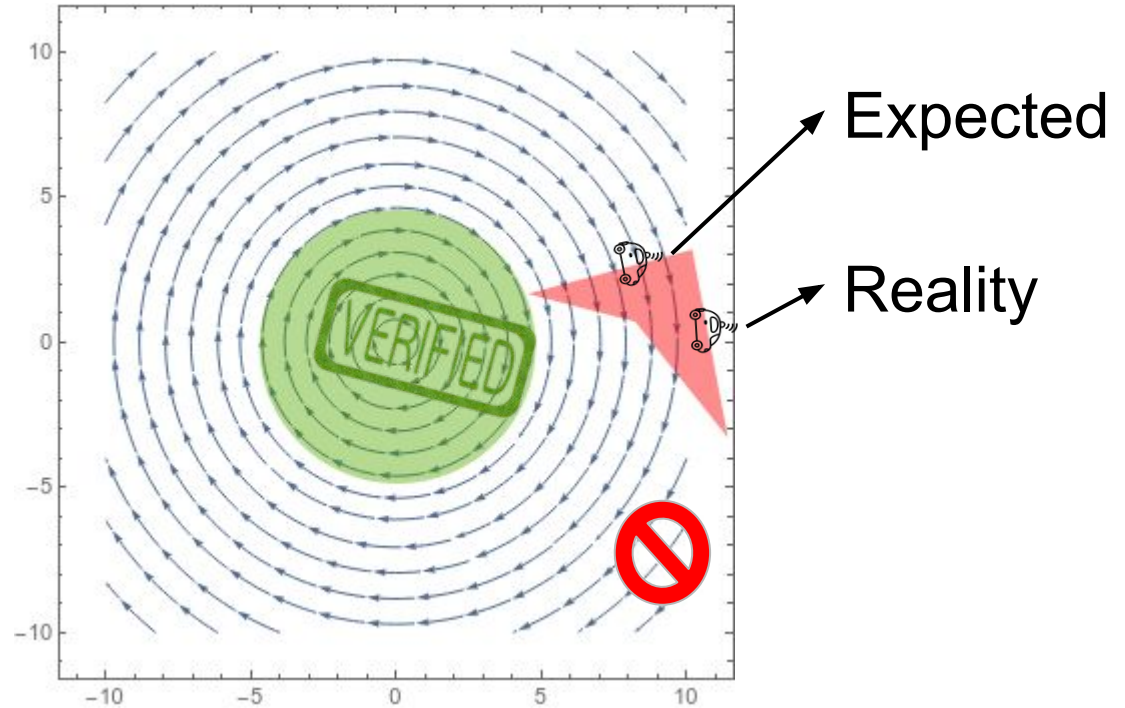
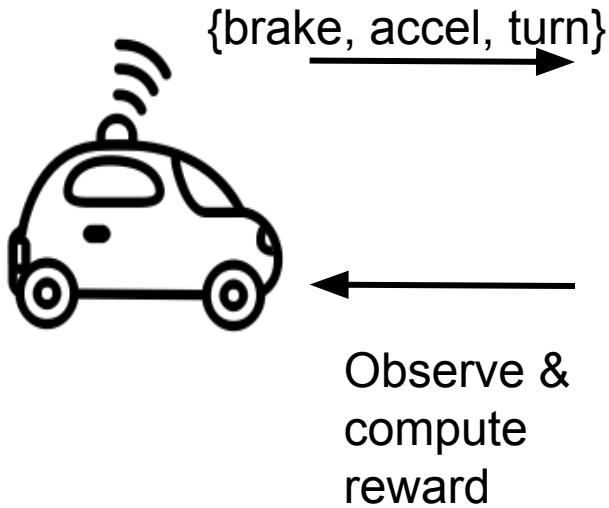


Model is correct.

Model is inaccurate

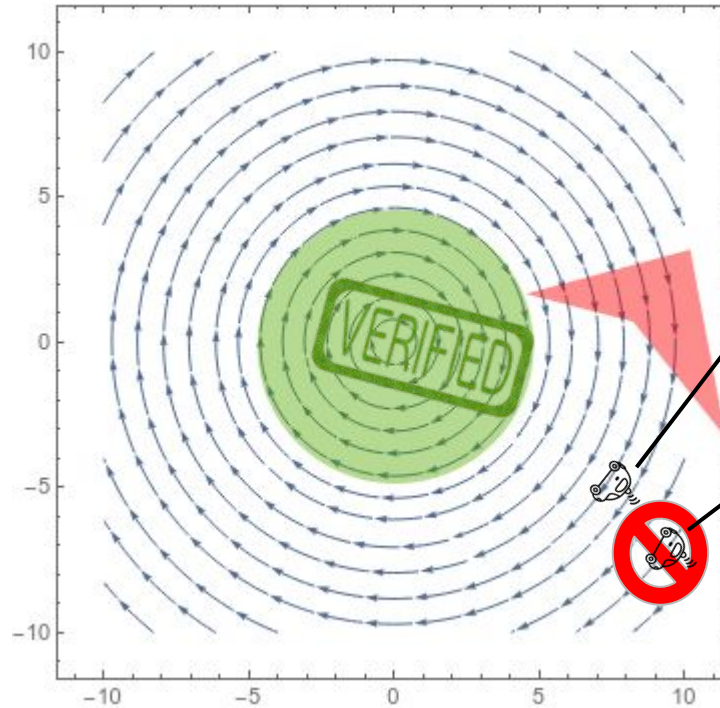
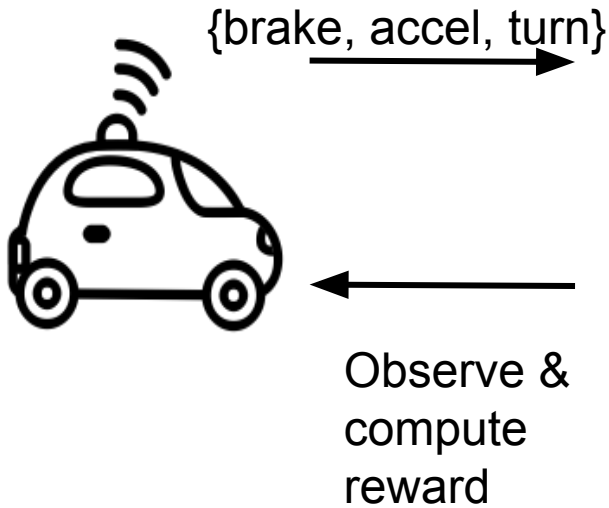
Obstacle!

# What About the Physical Model?





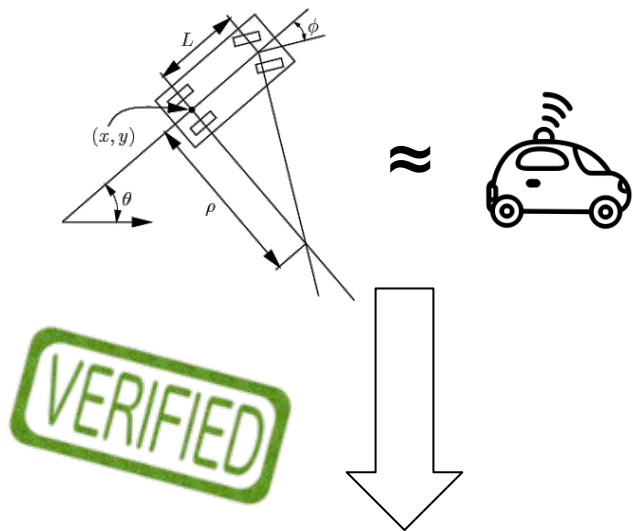
# What About the Physical Model?



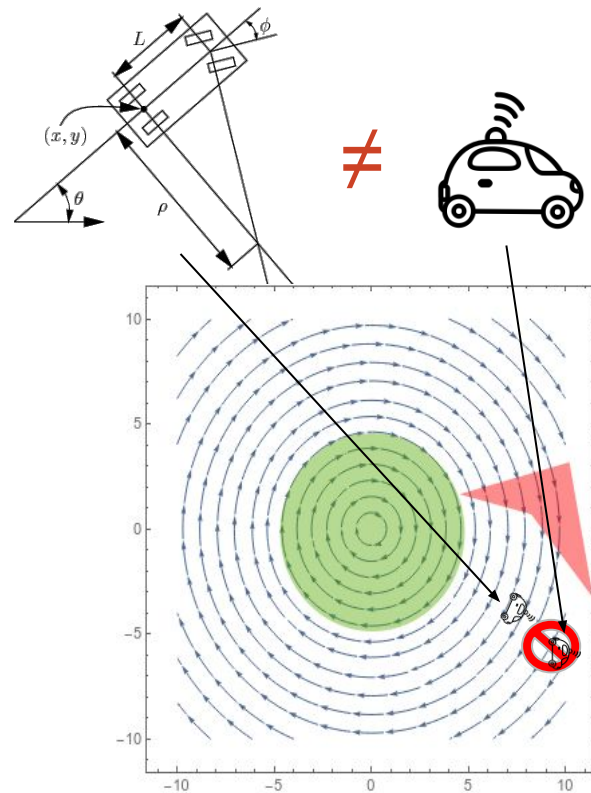
Expected  
(safe)

Reality  
(crash!)

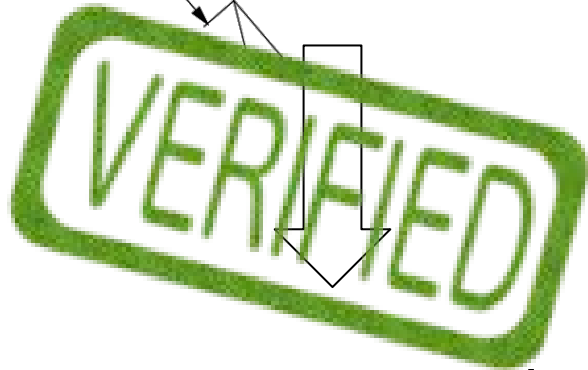
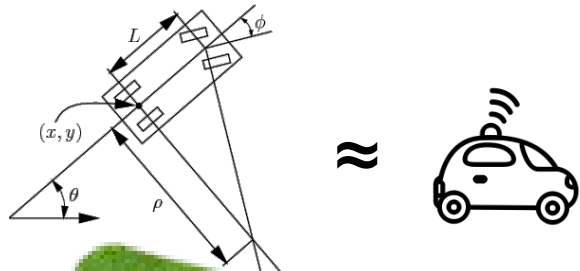
# Justified Speculative Control



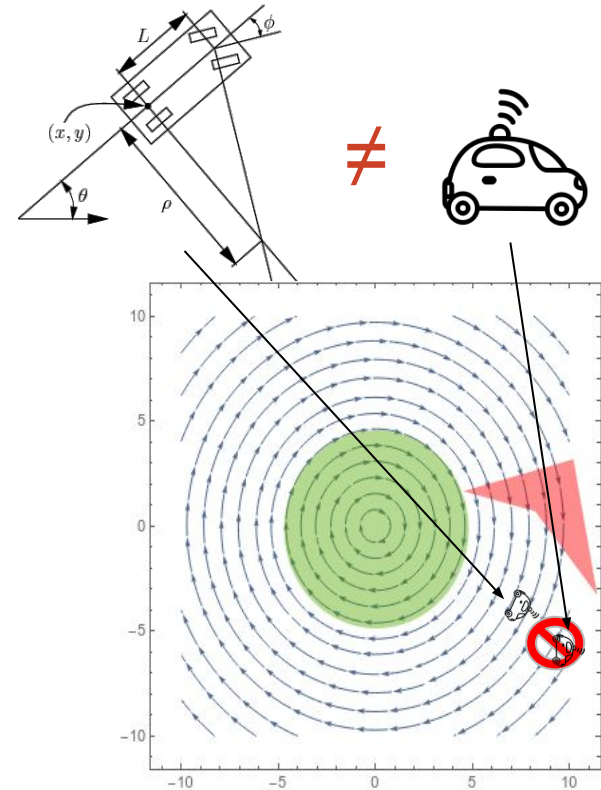
Learn over a constrained  
action space



# Justified Speculative Control



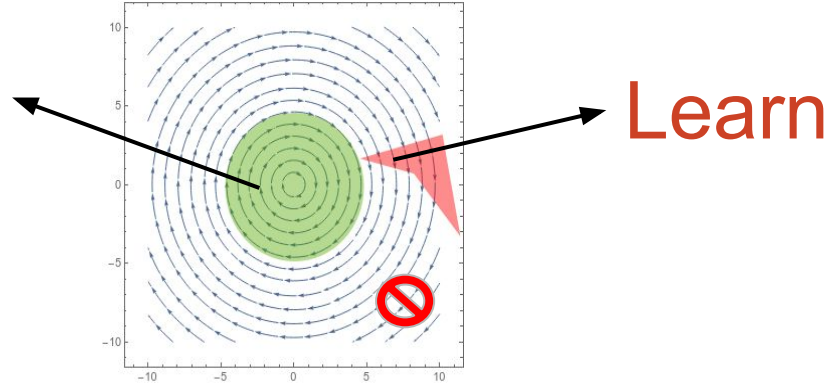
Learn over a constrained  
action space



# Justified Speculative Control

Learn over a  
constrained action  
space

VERIFIED



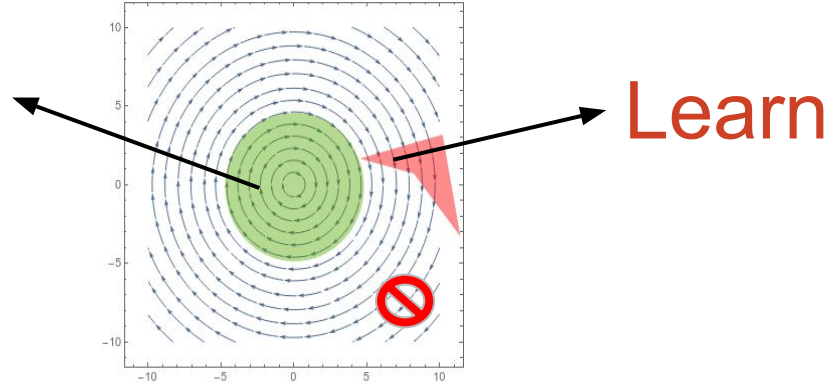
Some Questions:

1. How do we **know** when we're in unmodeled state space?
2. What do we **do** when we *are* in modeled state space?

# Justified Speculative Control

Learn over a  
constrained action  
space

VERIFIED



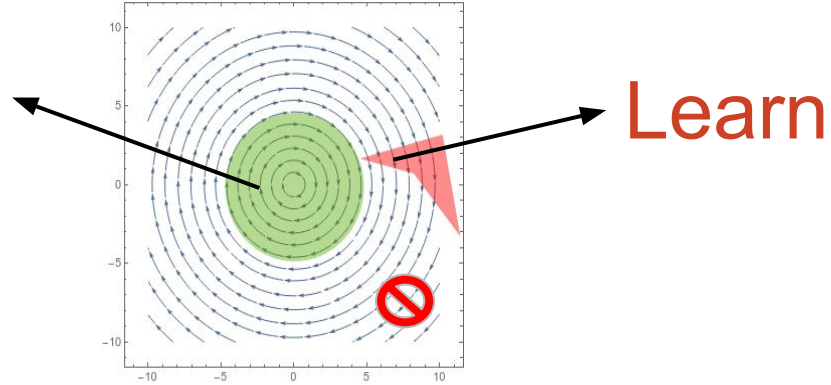
Some Questions:

1. How do we **know** when we're in unmodeled state space?
2. What do we **do** when we *are* in modeled state space?

# Justified Speculative Control

Learn over a  
constrained action  
space

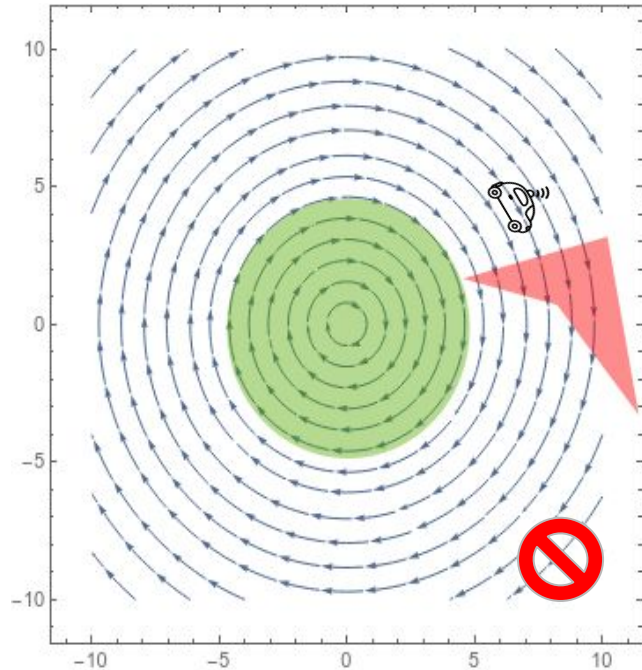
VERIFIED



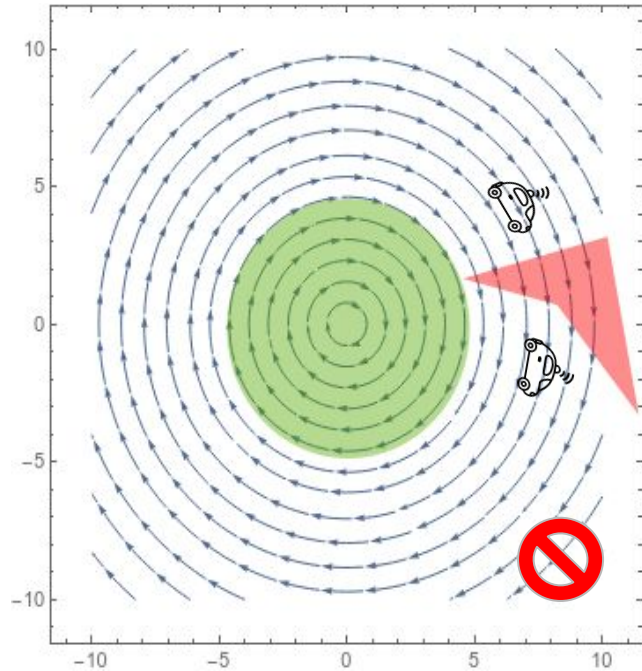
Theorem: Verification results are preserved outside of red region. But:

- How do we know when we're in unmodeled state space?
- What do we do when we *are* in modeled state space?**

# What do we do in unmodeled state-space?

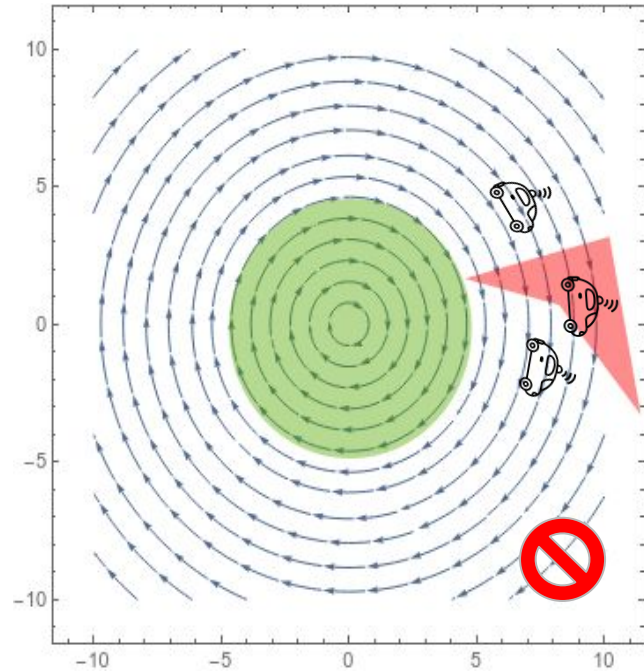


# What do we do in unmodeled state-space?

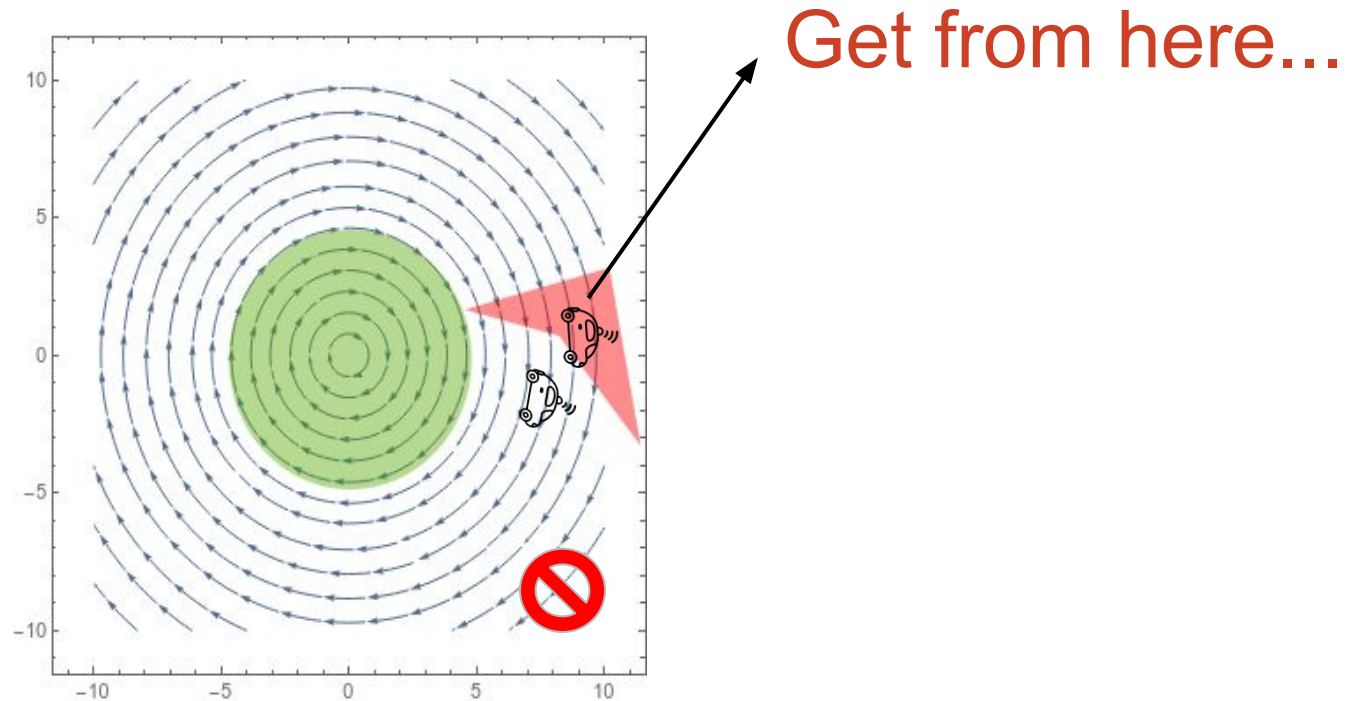




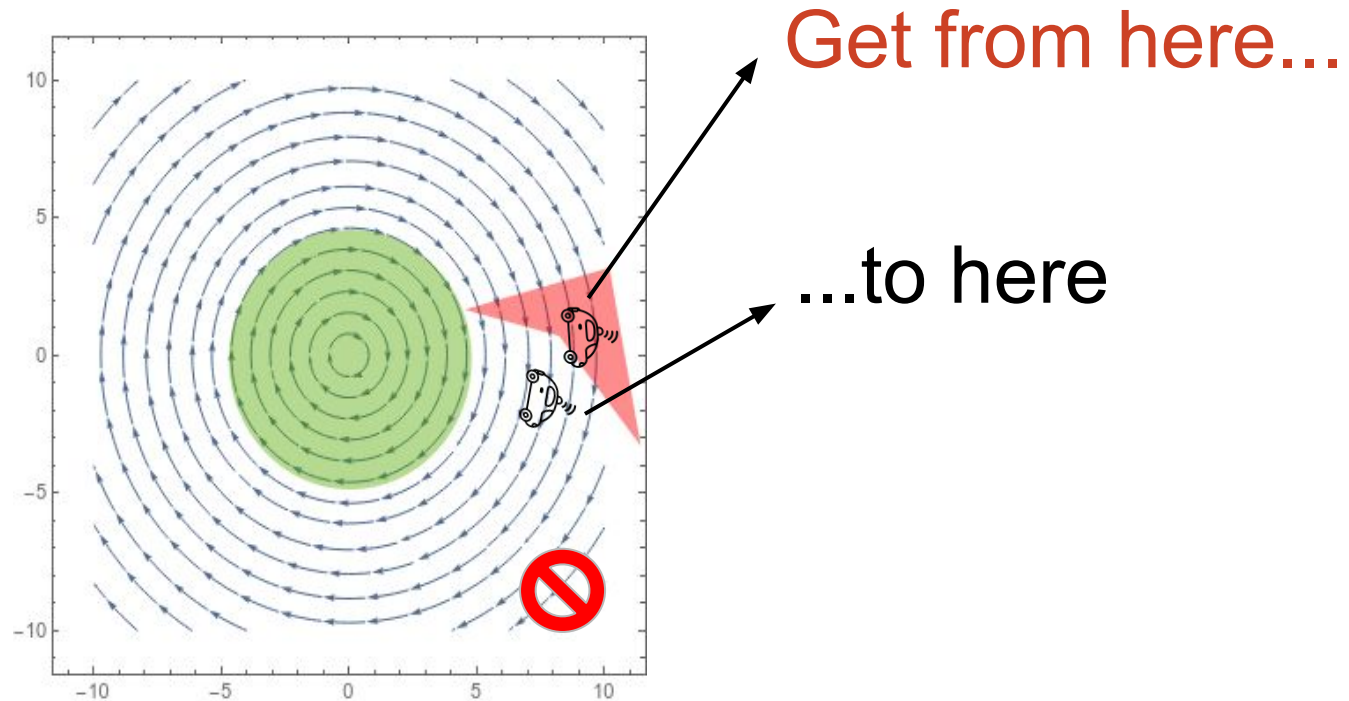
# What do we do in unmodeled state-space?



# What do we do in unmodeled state-space?



# What do we do in unmodeled state-space?



# Leveraging Formal Methods during Learning



Own Car



Leader

# Leveraging Formal Methods during Learning



Own Car



Leader

Perturbation	“Don’t hit the leader”	“Get back to modeled state space”
5%	3	2
25%	18	16
50%	41	24

# Conclusion

KeYmaera X + Justified Speculative Control:

1. Transfer **formal** verification results for **non-deterministic** control policies to policies obtained via a generic reinforcement learning algorithm.



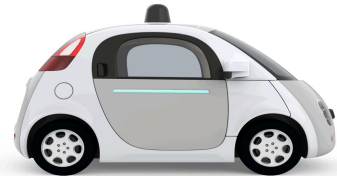
# Conclusion

KeYmaera X + Justified Speculative Control:

1. Transfer **formal** verification results for **non-deterministic** control policies to policies obtained via a generic reinforcement learning algorithm.
2. Leverages insights obtained during verification to direct future learning.



≠



# Model-Based Verification

# Reinforcement Learning



```
init → [{  
    {?safeAccel; accel  
    U brake};  
    t:=0; {pos'=vel, vel'=acc}  
}*](pos < stopSign)
```